Analysis and Control of Large Scale Aerospace Systems
from Planetary Landing to Drone Traffic Management

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Motivation: Drone Traffic Management

Controlling A Drone

Controlling Swarm of Drones
Motivation: Drone Traffic Management

Controlling A Drone

Controlling Swarm of Drones

Large number of agents $\sim$ Population density
Motivation: Mars Entry-Descent-Landing

- Navigational uncertainty
- Heating uncertainty
- Chute deployment uncertainty
- Landing footprint uncertainty

Image credit: NASA JPL
Motivation: Mars Entry-Descent-Landing

Large number of uncertain scenarios \(\sim\) Probability density
Motivation: Mars Entry-Descent-Landing

Supersonic parachute

Gale Crater (4.49S, 137.42E)
What to Analyze and Control

Signal

Sets

Densities
Outlook

Continuum of systems

Finitely many systems

One system
Outline of Today’s Talk

Part I: An Application
Propagating Density in Planetary EDL

Part II: A Theory
Controlling Density

Part III: Ongoing and Future Research
Unmanned Aerial Systems Traffic Management
Part I. An Application

Propagating Density in Planetary EDL
Forecasting, Estimation, Validation, Verification

Joint work with R. Bhattacharya (Texas A&M), J. Balaram (JPL)
State-of-the-art

Nonlinear Dynamics with Monte Carlo on Samples

Linear Dynamics with Gaussian Uncertainty

too expensive for EDL simulation
too ideal for EDL simulation
State-of-the-art

Nonlinear Dynamics with Monte Carlo on Samples

Linear Dynamics with Gaussian Uncertainty

too expensive for EDL simulation
too ideal for EDL simulation
How Bad is Gaussian Fit

Source: Golombek et. al., J. Geophys. Research. 2003

Credit: NASA JPL, Univ. Washington, St. Louis, JHU APL, Univ. Arizona.
Propagating Joint Density Function

Trajectory dynamics

\[ \dot{x}(t) = f(x(t), p), \quad x \in \mathbb{R}^{n_s}, \quad p \in \mathbb{R}^{n_p}; \quad x(0), p \text{ random} \]

\[ \dot{x}_e(t) = f_e(x_e(t)), \quad x_e := \begin{bmatrix} x \\ p \end{bmatrix} \in \mathbb{R}^{n_s+n_p}, \quad x_e(0) \sim \rho_0(x_e) \]

Density dynamics

Liouville PDE for joint density \( \rho(x_e(t), t) \)

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho f) = 0 \]

Method of characteristics (MOC)

\[ \dot{x}_e(t) = f_e(x_e(t)), \quad \dot{\rho}(t) = -\rho \nabla \cdot f, \quad \begin{bmatrix} x_e(0) \\ \rho(0) \end{bmatrix} = \begin{bmatrix} x_e(0) \\ \rho_0(x_e(0)) \end{bmatrix} \]
MOC
PDE BVP $\xrightarrow{\text{MOC}}$ ODE IVP

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Application to Mars EDL
Landing Footprint Uncertainty
Application to Mars EDL
Chute Deployment Uncertainty


Extension for Process Noise

Trajectory Dynamics is Stochastic Differential Equation

Process Noise

KL expansion

Nonparametric → Parametric

Fokker-Planck PDE (2nd order) → Liouville PDE (1st order)

Function approximation

\[ \rho(x(t), t) \] \[ N \to \infty \] \[ \rho_N(x_N(t), t) \]

Application to Nonlinear Filtering

$\mathcal{W}_2 \left( \xi_{\text{Benes}}^+ (t), \xi_{\text{Particle}}^+ (t) \right)$

$\mathcal{W}_2 \left( \xi_{\text{Benes}}^+ (t), \xi_{\text{KLMOC}}^+ (t) \right)$


A.H., L. Lee, R. Bhattacharya, A Dynamical System Pair with Identical First Two Moments But Different Probability Densities, *CDC*, 2014.


F-16 Flight Controller Verification

LQR vs. gsLQR Results: (MC)

Model Refinement

A.H., R. Bhattacharya, Geodesic Density Tracking with Applications to Data Driven Modeling, ACC, 2014.
Part II. A Theory

Controlling Density

Finite Horizon LQG Density Regulator

Joint work with E.D.B. Wendel (Draper Laboratory)
How to Go from One Density to Another
or Close to Another

\[ x(0) \sim \rho_0 \]

\[ x(t_1) \sim \rho_1 \]
LQG State Regulator

\[
\min_{u \in U} \phi (x_1, x_d) + \mathbb{E}_x \left[ \int_0^{t_1} (x^\top Q x + u^\top Ru) \, dt \right]
\]

\[
dx(t) = A x(t) \, dt + B u(t) \, dt + F \, dw(t),
\]

\[
x(0) = x_0 \text{ given}, \quad x_d \text{ given}, \quad t_1 \text{ fixed},
\]

Typical terminal cost: MSE

\[
\phi (x_1, x_d) = \mathbb{E}_{x_1} \left[ (x_1 - x_d)^\top M (x_1 - x_d) \right]
\]
LQG Density Regulator

$$\min_{u \in U} \varphi (\rho_1, \rho_d) + \mathbb{E}_x \left[ \int_0^{t_1} (x^\top Q x + u^\top R u) \, dt \right]$$

$$dx(t) = Ax(t) \, dt + Bu(t) \, dt + F \, dw(t),$$

$$x(0) \sim \rho_0 \text{ given, } x_d \sim \rho_d \text{ given, } t_1 \text{ fixed,}$$

Proposed terminal cost: MMSE

$$\varphi (x_1, x_d) = \inf_{y \sim \rho \in P_2(\rho_1, \rho_d)} \mathbb{E}_y \left[ (x_1 - x_d)^\top M (x_1 - x_d) \right],$$

where $y := (x_1, x_d)^\top$
Formulation: LQG Density Regulator

\[
\begin{align*}
\phi(\rho_1, \rho_d) \\
\min_{u \in \mathcal{U}} \inf_{y \sim \rho \in \mathcal{P}_2(\rho_1, \rho_d)} & \mathbb{E}_y \left[ (x_1 - x_d)^\top M (x_1 - x_d) \right] \\
& + \mathbb{E}_x \left[ \int_0^{t_1} (x^\top Q x + u^\top R u) \, dt \right] \\
\end{align*}
\]

\[
dx(t) = Ax(t) \, dt + Bu(t) \, dt + F \, dw(t),
\]

\[
x(0) \sim \rho_0 = \mathcal{N} (\mu_0, S_0), \quad x_d \sim \rho_d = \mathcal{N} (\mu_d, S_d),
\]

\[
t_1 \text{ fixed,} \quad \mathcal{U} = \{u : u(x, t) = K(t)x + v(t)\}\]
\[
\infty \text{ dim. TPBVP } \leadsto (n^2 + 3n) \text{ dim. TPBVP}
\]

\[
\begin{pmatrix}
\dot{\mu}(t) \\
\dot{z}(t)
\end{pmatrix} = \begin{pmatrix}
A & BR^{-1}B^\top \\
Q & -A^\top
\end{pmatrix} \begin{pmatrix}
\mu(t) \\
z(t)
\end{pmatrix},
\]

\[
\dot{S}(t) = (A + BK^o)S(t) + S(t)(A + BK^o)^\top + FF^\top,
\]

\[
\dot{P}(t) = -A^\top P(t) - P(t)A - P(t)BR^{-1}B^\top P(t) + Q,
\]

**Boundary conditions:**

\[
\mu(0) = \mu_0, \ z(t_1) = M(\mu_d - \mu_1),
\]

\[
S(0) = S_0, \ P(t_1) = \left( S_d^{\frac{1}{2}} \left( S_d^{\frac{1}{2}} S_1^{-1} S_d^{\frac{1}{2}} \right)^{\frac{1}{2}} S_d^{\frac{1}{2}} - I_n \right) M
\]
Controlled State Covariance

\[ \rho_d = \mathcal{N}(\mu_d, S_d) \]

\[ \rho_1 = \mathcal{N}(\mu_1, S_1) \]
Expected Optimal Control

\[ E[u^o(t)] \]

UTM

Unmanned Aerial Systems Traffic Management
Vision for UAS Traffic Management (UTM)

Class G airspace extends up to 1200 ft AGL

500 ft AGL

Weight no more than 55 lbs

200 ft AGL

Requires: Automated V2V separation management
Yield manned traffic
Avoid obstacles (buildings, towers etc.)
Technical Challenges

Dynamic Geofencing

Control over LTE

Wind Uncertainty

Provable Safety

Image credit: NASA Ames Research Center
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Offline Protocol

How FAA approves a flight path request?
Offline Protocol
Path Planning and Deconfliction

requested flight paths

Server

Path planner 1

UAS 1
Offline Protocol
Path Planning and Deconfliction

{Wind, geofencing, obstacle} database forecasts

Server requested flight paths

Path planner 1 suggested flight paths

UAS 1
Offline Protocol
Path Planning and Deconfliction

Server
Path planner 1
UAS 1
requested
flight paths
{Wind, geofencing,
obstacle} database
forecasts
suggested
flight paths
Path manager 1
Drone team 1
commanded paths
Approved flight 
path history
update database
update history
{Wind, geofencing,
obstacle} database
accepted paths, 
drone license IDs
update 
database
update 
history
Path planner 1
suggested 
flight paths
Server
requested 
flight paths
Path manager 1
Drone team 1
commanded paths
Offline Protocol
Path Planning and Deconfliction

- Approved flight path history
  - update database
  - update history

- {Wind, geofencing, obstacle} database

- Server
  - forecasts
  - requested flight paths
  - accepted paths, drone license IDs

- Path planner 1
  - suggested flight paths
  - commanded paths
  - Path manager 1
  - Drone team 1

- Path planner 2
  - suggested flight paths
  - commanded paths
  - Path manager 2
  - Drone team 2

- UAS 1
  - requested flight paths
  - accepted paths, drone license IDs

- UAS 2
  - requested flight paths
  - accepted paths, drone license IDs
Motion Protocol

What does an individual drone do in real time?
Input: Approved Flight Path
Reach Set Evolution due to Wind Uncertainty

$\mathcal{R}_{\pi_F} (\{O\}, t_f - t_0)$
Discrete Decision Making Instances

\[ \mathcal{R}_{\pi_F}(\{O\}, t_f - t_0) \]
4D Flight Tubes $\mathcal{F}_{[t_j, t_{j+1}]}$

Reach set enclosed with safety annulus

$\mathcal{F}_{[t_0, t_1]}$, $\mathcal{F}_{[t_1, t_2]}$
4D Flight + Landing Tubes \( \{ \mathcal{F}_{[t_j, t_{j+1}]}, \mathcal{L}_{[t_{j+1}, t_{j+2}]} \} \)
Motion Protocol: $t = t_0$

**IF:** Have all + ACKs for $\{F_{[t_0,t_1)}, L_{[t_1,t_2)}\}$ AND $D \in R_{\pi_F}(\{O\}, t_f - t_0)$

**THEN:** Take-off AND broadcast req. for $\{F_{[t_1,t_2)}, L_{[t_2,t_3)}\}$
Motion Protocol: \( t \in [t_0, t_1) \)

Listening for ± ACKs, \( x(t) \in F_{[t_0, t_1)} \)
**Motion Protocol:** \( t = t_1 \)

**IF:** All + ACKs AND \( D \in \mathcal{R}_{\pi_F} (\{x(t_1)\}, t_f - t_1) \)

**THEN:** Continue in \( \mathcal{F}_{[t_1, t_2)} \) AND broadcast req. for \( \{\mathcal{F}_{[t_2, t_3)}, \mathcal{L}_{[t_3, t_4]}\} \)

**ELSE:** Abort mission via \( \mathcal{L}_{[t_1, t_2)} \)
Motion Protocol: $t = t_1$

**IF:** All + ACKs AND $D \in \mathcal{R}_{\pi_F}(\{x(t_1)\}, t_f - t_1)$

**THEN:** Continue in $\mathcal{F}_{[t_1,t_2]}$ AND broadcast req. for $\{\mathcal{F}_{[t_2,t_3]}, \mathcal{L}_{[t_3,t_4]}\}$

**ELSE:** Abort mission via $\mathcal{L}_{[t_1,t_2]}$
Motion Protocol: $t = t_1$

**IF:** All + ACKs AND $D \notin \mathcal{R}_{\pi_F}(\{x(t_1)\}, t_f - t_1)$

**THEN:** Continue in $\mathcal{F}_{[t_1, t_2]}$ AND broadcast req. for $\{\mathcal{F}_{[t_2, t_3]}, \mathcal{L}_{[t_3, t_4]}\}$

**ELSE:** Abort mission via $\mathcal{L}_{[t_1, t_2]}$
Algorithms for Motion Protocol

Compute minimum volume outer ellipsoids: SDP
Proposed Architecture: Performance

Throughput

$w_1(t)$ and $w_2(t)$ are different wind trajectories

Number of offline approvals

Too few take-offs

Too many aborts
Continuum of systems

Finitely many systems

One system
Thank You
Backup Slides for Part I
Backup Slides for Part II
$\varphi(N(\mu_1, S_1), N(\mu_d, S_d))$ equals

$$(\mu_1 - \mu_d)^\top M (\mu_1 - \mu_d) +$$

$$\min_{C \in \mathbb{R}^{n \times n}} \text{tr} ((S_1 + S_d - 2C)M) \text{ s.t. } \begin{bmatrix} S_1 & C \\ C^\top & S_d \end{bmatrix} \succeq 0$$
\( \varphi(\mathcal{N}(\mu_1, S_1), \mathcal{N}(\mu_d, S_d)) \) equals

\[
(\mu_1 - \mu_d)^\top M (\mu_1 - \mu_d) + \\
\min_{C \in \mathbb{R}^{n \times n}} \operatorname{tr}((S_1 + S_d - 2C)M) \quad \text{s.t.} \quad \begin{bmatrix} S_1 & C \\ C^\top & S_d \end{bmatrix} \succeq 0
\]

\[\upuparrows\]

\[
\max_{C \in \mathbb{R}^{n \times n}} \operatorname{tr}(CM) \quad \text{s.t.} \quad S_1 - CS_d^{-1}C^\top \succeq 0
\]

\[\upuparrows\]

\[
C^* = S_1 S_d^{\frac{1}{2}} \left( S_d^{\frac{1}{2}} S_1 S_d^{\frac{1}{2}} \right)^{-\frac{1}{2}} S_d^{\frac{1}{2}}
\]
This gives

\[ \varphi \left( \mathcal{N} (\mu_1, S_1), \mathcal{N} (\mu_d, S_d) \right) = (\mu_1 - \mu_d)^\top M (\mu_1 - \mu_d) \]

\[ + \text{tr} \left( MS_1 + MS_d - 2 \left[ (\sqrt{S_d} MS_1 \sqrt{S_d}) (\sqrt{S_d} S_1 \sqrt{S_d})^{-1} \right] \right) \]

Applying maximum principle:

\[ K^0(t) = R^{-1} B^\top P(t), \]

\[ \nu^0(t) = R^{-1} B^\top (z(t) - P(t) \mu(t)) \]
Matrix Geometric Mean

The minimal geodesic $\gamma^* : [0, 1] \mapsto S^+_n$ connecting $\gamma(0) = S_d$ and $\gamma(1) = S_1^{-1}$, associated with the Riemannian metric $g_A(S_d, S_1^{-1}) = \text{tr} (A^{-1} S_d A^{-1} S_1^{-1})$, is

$$\gamma^*(t) = S_d \#_t S_1^{-1} = S_d^{\frac{1}{2}} \left( S_d^{-\frac{1}{2}} S_1^{-1} S_d^{-\frac{1}{2}} \right)^t S_d^{\frac{1}{2}}$$

$$= S_1^{-1} \#_{1-t} S_d = S_1^{-\frac{1}{2}} \left( S_1^{\frac{1}{2}} S_d S_1^{\frac{1}{2}} \right)^{1-t} S_1^{-\frac{1}{2}}$$

Geometric Mean:

$$\gamma^* \left( \frac{1}{2} \right) = S_d \#_{\frac{1}{2}} S_1^{-1} = S_1^{-1} \#_{\frac{1}{2}} S_d$$
Example

\[
\begin{pmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{pmatrix}
= \begin{bmatrix}
0 & 1 \\
2 & -3
\end{bmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
dt + \begin{bmatrix}
0 \\
1
\end{bmatrix}
u
dt + \begin{bmatrix}
0.01 \\
0.01
\end{bmatrix}
dw
\]

\[\rho_0 = \mathcal{N} \left( (1, 1)^\top, I_2 \right), \quad \rho_d = \mathcal{N} \left( (0, 0)^\top, 0.1 \ I_2 \right),\]

\[Q = 100 \ I_2, \quad R = 1, \quad M = I_2, \quad t_1 = 2\]
Backup Slides for Part III
Input-Output for Motion Protocol

{Drone_ID}_k, F_{t_jk,t_{j_k+1}}, L_{t_jk,t_{j_k+2}} \}

{A, A, ?, A, ?, A, D, A, ?}

Communication Protocol

{Drone_ID}_\ell, F_{t_{j_\ell},t_{j_\ell+1}}, L_{t_{j_\ell+1},t_{j_\ell+2}} \}

{A}

Communication Protocol