Control of Large Scale Cyberphysical Systems

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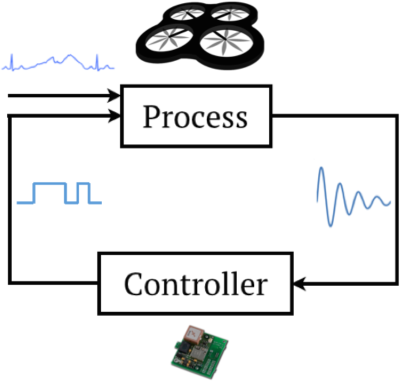
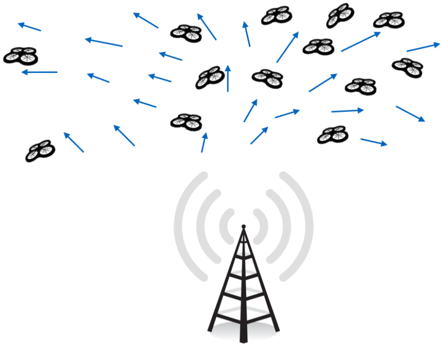
Motivation: Drone Traffic Management

Controlling A Drone

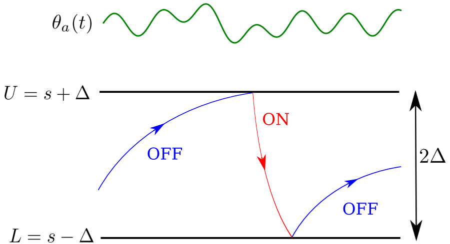


Controlling Swarm of Drones

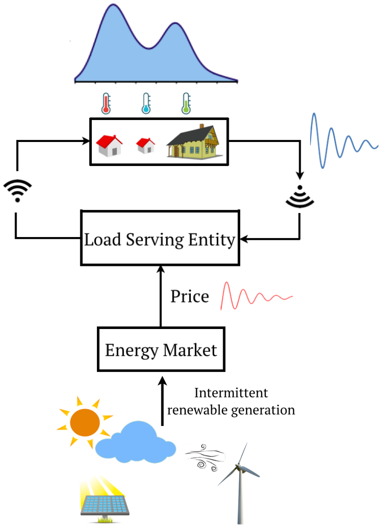


Motivation: Smart Grid Demand Response

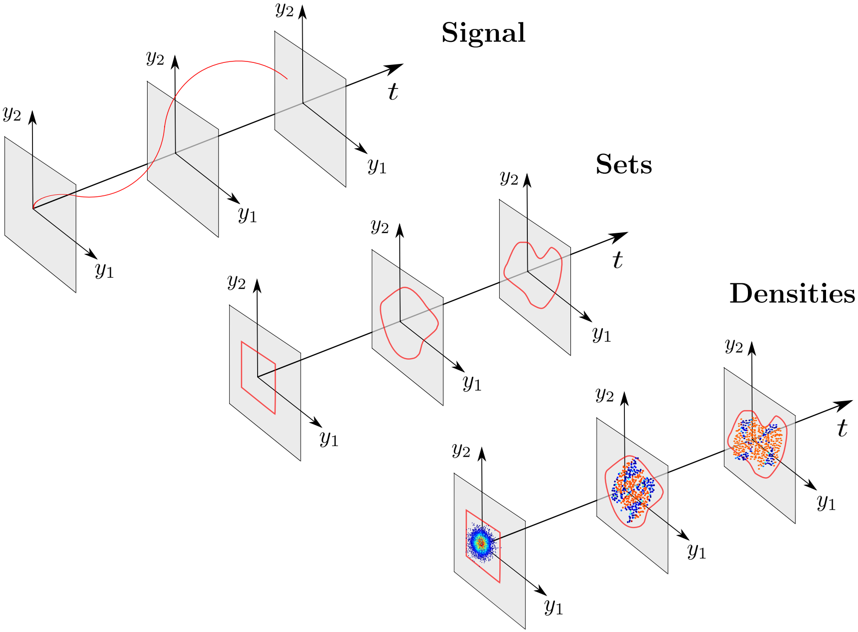
Controlling An AC



Controlling Population of ACs



What to Control

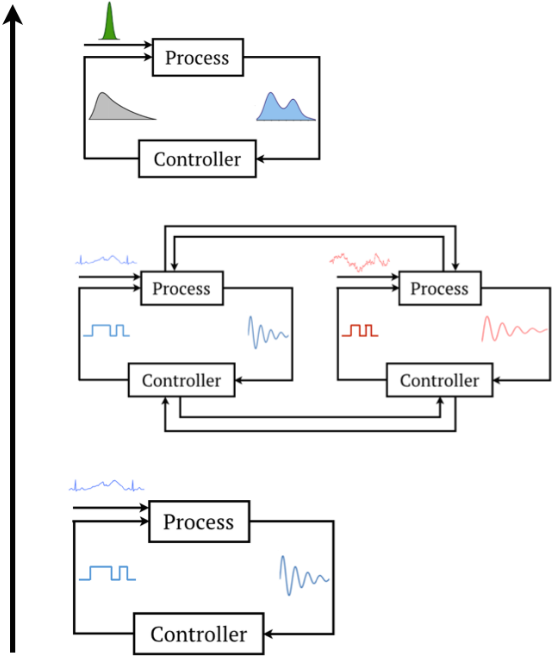


Outlook

Continuum of systems

Finitely many systems

One system



Outline of Today's Talk

Part I: An Application

Controlling Air Conditioners

Part II: A Theory

Controlling Density

Part III: Ongoing and Future Research

Unmanned Aerial Systems Traffic Management

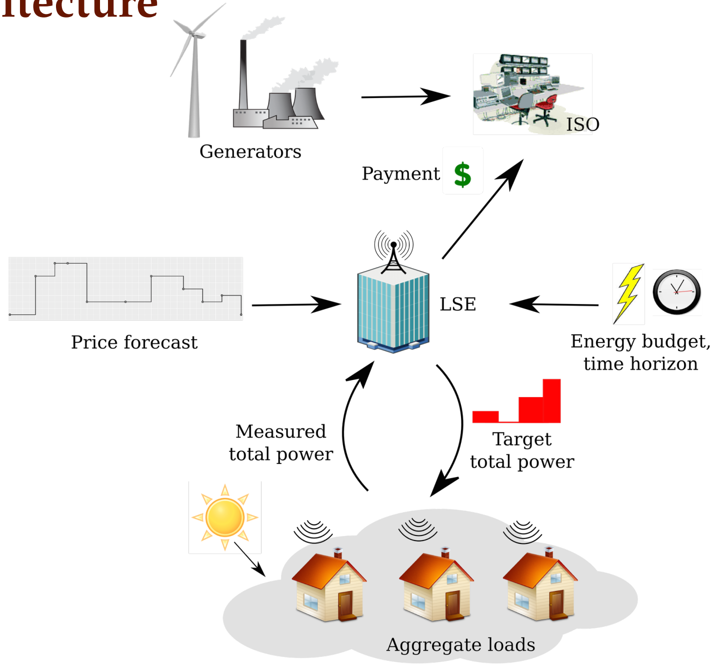
Part I. An Application

Controlling Air Conditioners

Direct Control for Demand Response

Joint work with X. Geng, F.A.C.C. Fontes, P.R. Kumar, and L. Xie

Architecture



Research Scope

Objective: A theory of operation for the LSE

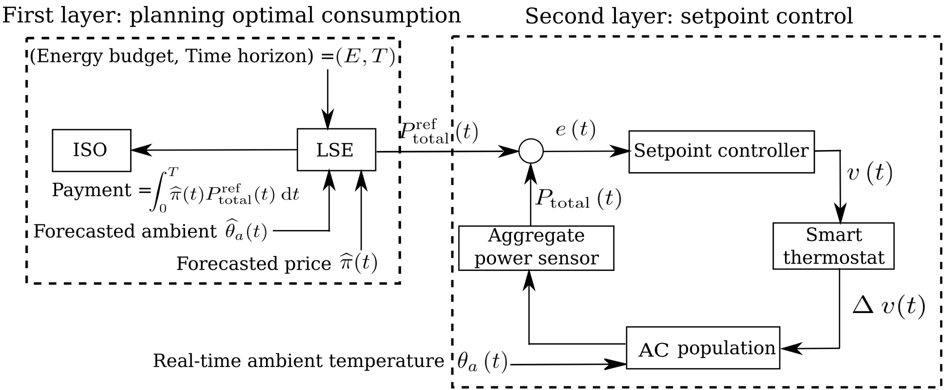
Challenges:

1. How to design the **target consumption as a function of price**?

2. How to control so as to preserve **privacy** of the loads' states?

3. How to respect loads' **contractual obligations** (e.g. comfort range width Δ)?

Two Layer Block Diagram



First Layer: Planning Optimal Consumption

price forecast

$$\underset{\{u_1(t),\ldots,u_N(t)\}\in\{0,1\}^N}{\text{minimize}} \quad \int_0^T \frac{P}{\eta} \, \widehat{\pi}(t) \, (u_1(t)+u_2(t)+\ldots+u_N(t)) \, \mathrm{d}t$$

subject to

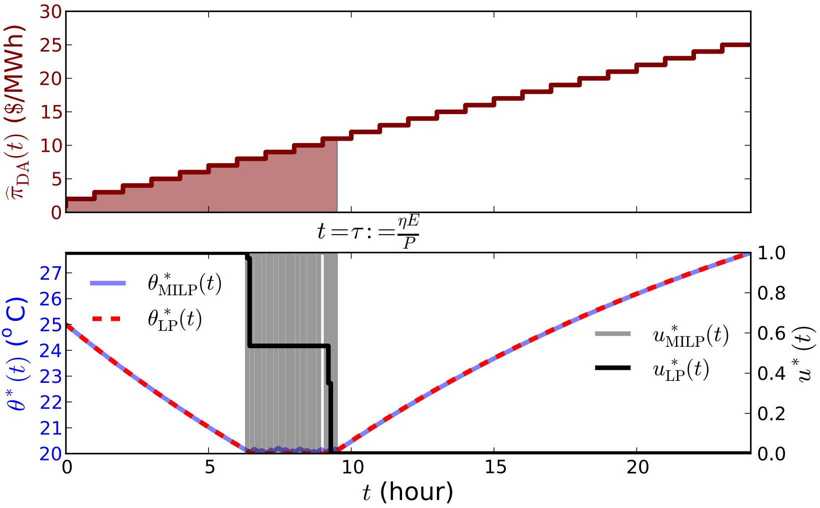
$$(1) \quad \dot{\theta}_i = -\alpha_i \left(\theta_i(t) - \widehat{\theta}_a(t)\right) - \beta_i P u_i(t) \qquad \forall\, i = 1,\ldots,N,$$

$$(2) \quad \int_0^T (u_1(t)+u_2(t)+\ldots+u_N(t)) \, \mathrm{d}t = \tau \doteq \frac{\eta E}{NP} (< T, \text{given})$$

$$(3) \quad L_{i0} \le \theta_i(t) \le U_{i0} \qquad \forall\, i = 1,\ldots,N.$$

Optimal consumption: $P^*_{\text{ref}}(t) = \frac{P}{\eta} \sum_{i=1}^{N} u_i^*(t)$

First Layer: Solution



Numerical challenges for MILP and LP

Solution: continuous time ⇝ PMP w. state inequality constraints

Second Layer: Real-time Setpoint Control

optimal reference | error | measured

$$\underbrace{P^*_{\text{ref}}(t)}_{\text{optimal reference}} = \frac{P}{\eta}\sum_{i=1}^{N} u_i^*(t), \rightsquigarrow \quad \underbrace{e(t)}_{\text{error}} = P^*_{\text{ref}}(t) - \underbrace{P_{\text{total}}(t)}_{\text{measured}},$$

PDE based velocity control | gain | broadcast

$$v(t) = \underbrace{\gamma_{\text{tracking}}\,(e(t))}_{\text{PDE based velocity control}}, \qquad \frac{\mathrm{d}s_i}{\mathrm{d}t} = \underbrace{\Delta_i}_{\text{gain}} \; \underbrace{v(t)}_{\text{broadcast}},$$

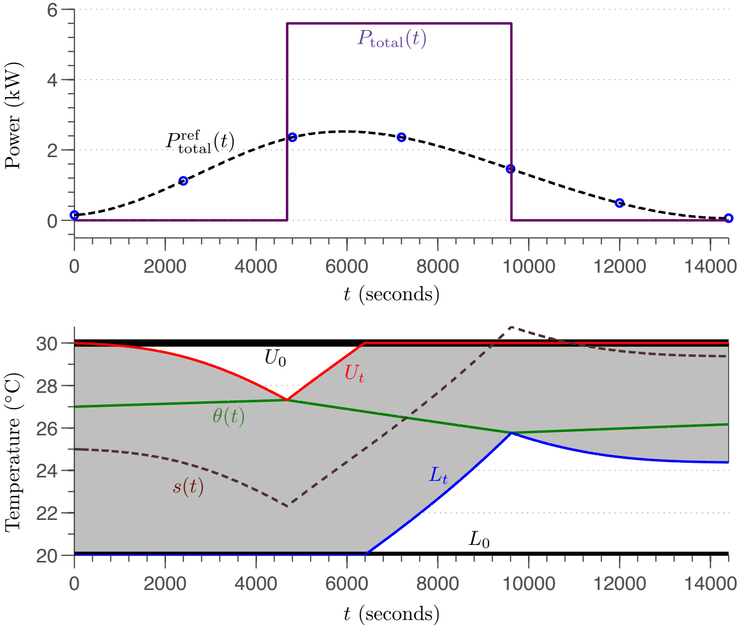
Moving lower boundary

$$L_{it} = U_{i0} \wedge \left[L_{i0} \vee (s_i(t) - \Delta_i)\right],$$

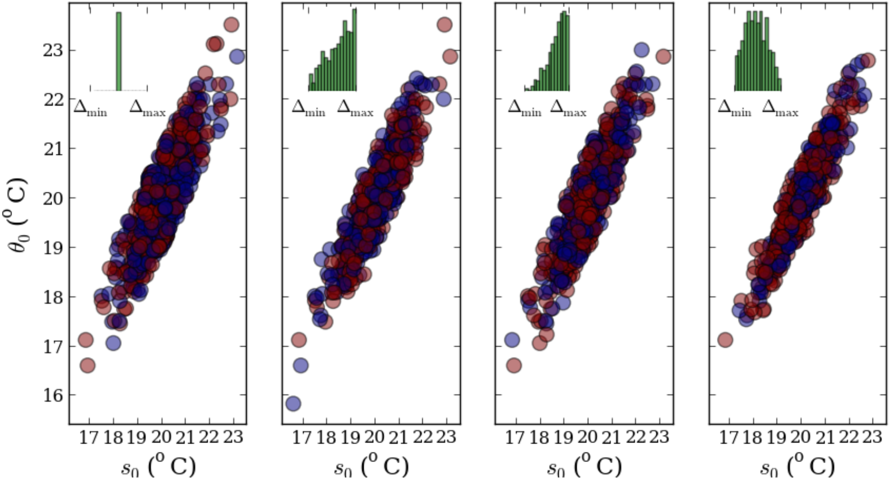
Moving upper boundary

$$U_{it} = L_{i0} \vee \left[U_{i0} \wedge (s_i(t) + \Delta_i)\right].$$

Boundary Control: Deadband → Liveband

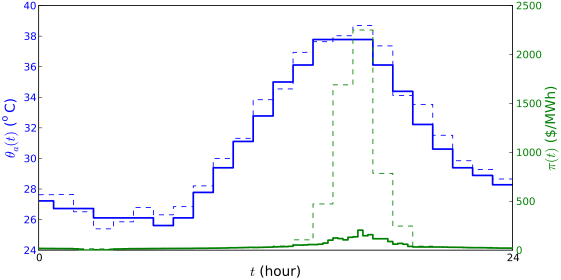


Initial Condition and Δ Distribution for 500 Homes

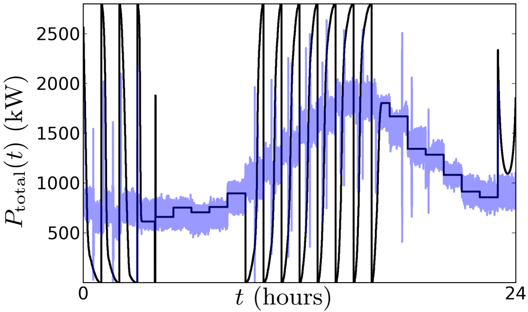


Houston Temperature + Market Price

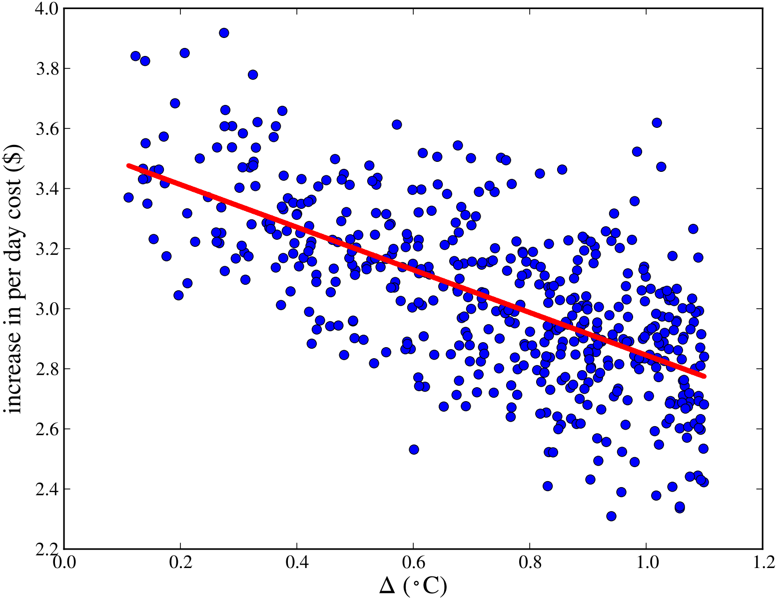
- forecasted ambient
- actual ambient
- forecasted price
- actual price



- target consumption
- actual consumption



How Can the LSE Price A Contract



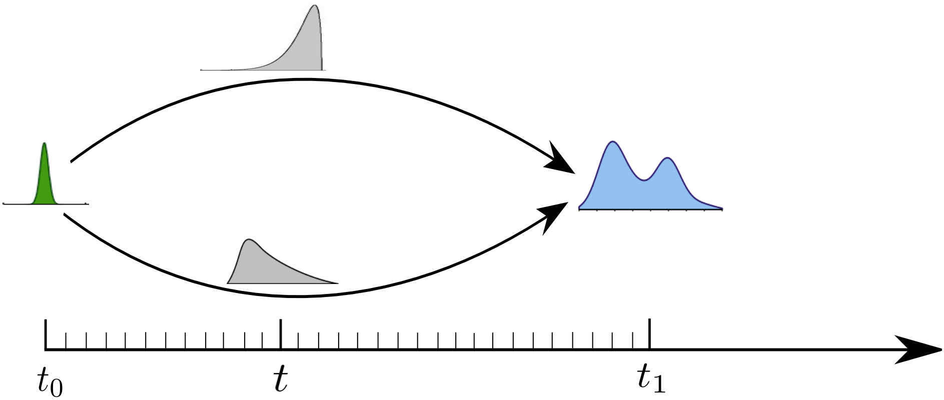
Part II. A Theory

Controlling Density

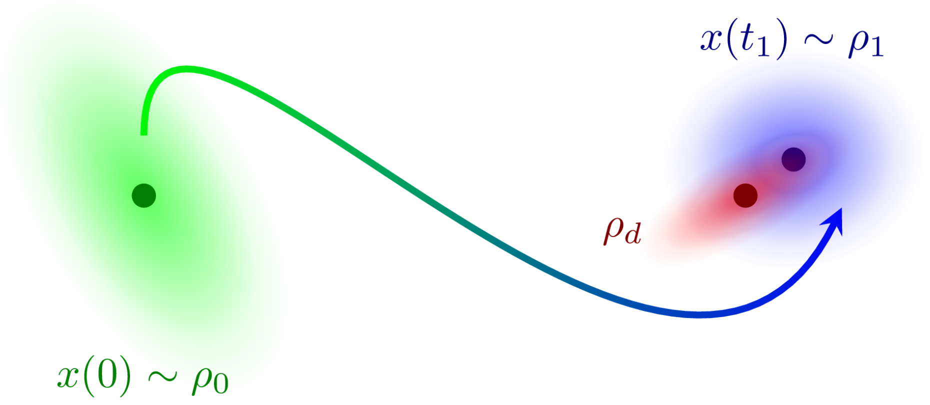
Finite Horizon LQG Density Regulator

Joint work with E.D.B. Wendel (Draper Laboratory)

How to Go from One Density to Another



or Close to Another



LQG State Regulator

$$\min_{u\in\mathcal{U}} \phi\left(x_1, x_d\right) \; + \; \mathbb{E}_x \left[\int_0^{t_1} \left(x^\top Q x \; + \; u^\top R u\right) \mathrm{d}t\right]$$

$$\mathrm{d}x(t) = Ax(t)\,\mathrm{d}t + Bu(t)\,\mathrm{d}t + F\,\mathrm{d}w(t),$$

$$x(0) = x_0 \text{ given}, \quad x_d \text{ given}, \quad t_1 \text{ fixed},$$

Typical terminal cost: MSE

$$\phi\left(x_1, x_d\right) = \mathbb{E}_{x_1}\left[(x_1 - x_d)^\top M (x_1 - x_d)\right]$$

LQG Density Regulator

$$\min_{u \in \mathcal{U}} \varphi\left(\rho_1, \rho_d\right) \; + \; \mathbb{E}_x\left[\int_0^{t_1} (x^\top Q x \; + \; u^\top R u)\, \mathrm{d}t\right]$$

$$\mathrm{d}x(t) = Ax(t)\,\mathrm{d}t + Bu(t)\,\mathrm{d}t + F\,\mathrm{d}w(t),$$

$$x(0) \sim \rho_0 \text{ given}, \quad x_d \sim \rho_d \text{ given}, \quad t_1 \text{ fixed},$$

Proposed terminal cost: MMSE

$$\varphi\left(x_1, x_d\right) = \inf_{y \sim \rho \in \mathcal{P}_2(\rho_1, \rho_d)} \mathbb{E}_y\left[(x_1 - x_d)^\top M (x_1 - x_d)\right],$$

where $y := (x_1, x_d)^\top$

Formulation: LQG Density Regulator

$$\varphi(\rho_1, \rho_d)$$

$$\min_{u \in \mathcal{U}} \inf_{y \sim \rho \in \mathcal{P}_2(\rho_1, \rho_d)} \mathbb{E}_y \left[(x_1 - x_d)^\top M (x_1 - x_d) \right]$$

$$+ \mathbb{E}_x \left[\int_0^{t_1} (x^\top Q x + u^\top R u) \, \mathrm{d}t \right]$$

$$\mathrm{d}x(t) = Ax(t) \, \mathrm{d}t + Bu(t) \, \mathrm{d}t + F \, \mathrm{d}w(t),$$

$$x(0) \sim \rho_0 = \mathcal{N}(\mu_0, S_0), \quad x_d \sim \rho_d = \mathcal{N}(\mu_d, S_d),$$

$$t_1 \text{ fixed}, \quad \mathcal{U} = \{u \, : \, u(x,t) = K(t)x + v(t)\}$$

However, $\varphi\left(\mathcal{N}\left(\mu_1, S_1\right), \mathcal{N}\left(\mu_d, S_d\right)\right)$ equals

$$\left(\mu_1 - \mu_d\right)^\top M \left(\mu_1 - \mu_d\right) +$$

$$\min_{C \in \mathbb{R}^{n \times n}} \operatorname{tr}\left((S_1 + S_d - 2C)M\right) \text{ s.t. } \begin{bmatrix} S_1 & C \\ C^\top & S_d \end{bmatrix} \succeq 0$$

However, $\varphi\left(\mathcal{N}\left(\mu_1, S_1\right), \mathcal{N}\left(\mu_d, S_d\right)\right)$ equals

$$\left(\mu_1 - \mu_d\right)^\top M \left(\mu_1 - \mu_d\right) +$$

$$\min_{C \in \mathbb{R}^{n \times n}} \operatorname{tr}\left(\left(S_1 + S_d - 2C\right)M\right) \ \text{s.t.} \ \begin{bmatrix} S_1 & C \\ C^\top & S_d \end{bmatrix} \succeq 0$$

$$\Updownarrow$$

$$\max_{C \in \mathbb{R}^{n \times n}} \operatorname{tr}\left(CM\right) \quad \text{s.t.} \quad S_1 - C S_d^{-1} C^\top \succeq 0$$

$$\Updownarrow$$

$$C^* = S_1 S_d^{\frac{1}{2}} \left(S_d^{\frac{1}{2}} S_1 S_d^{\frac{1}{2}}\right)^{-\frac{1}{2}} S_d^{\frac{1}{2}}$$

This gives

$$\varphi\left(\mathcal{N}\left(\mu_1, S_1\right), \mathcal{N}\left(\mu_d, S_d\right)\right) = \left(\mu_1 - \mu_d\right)^\top M \left(\mu_1 - \mu_d\right)$$

$$+\mathrm{tr}\left(MS_1 + MS_d - 2\left[\left(\sqrt{S_d} M S_1 \sqrt{S_d}\right)\left(\sqrt{S_d} S_1 \sqrt{S_d}\right)^{-\frac{1}{2}}\right]\right)$$

Applying maximum principle:

$$K^o(t) = R^{-1} B^\top P(t),$$

$$v^o(t) = R^{-1} B^\top \left(z(t) - P(t)\mu(t)\right)$$

∞ dim. TPBVP $\rightsquigarrow$ $(n^2 + 3n)$ dim. TPBVP

$$\begin{pmatrix} \dot{\mu}(t) \\ \dot{z}(t) \end{pmatrix} = \begin{pmatrix} A & BR^{-1}B^\top \\ Q & -A^\top \end{pmatrix} \begin{pmatrix} \mu(t) \\ z(t) \end{pmatrix},$$

$$\dot{S}(t) = (A + BK^o)S(t) + S(t)(A + BK^o)^\top + FF^\top,$$

$$\dot{P}(t) = -A^\top P(t) - P(t)A - P(t)BR^{-1}B^\top P(t) + Q,$$

Boundary conditions:

$$\mu(0) = \mu_0, \quad z(t_1) = M(\mu_d - \mu_1),$$

$$S(0) = S_0, \quad P(t_1) = \left(S_d \# S_1^{-1} - I_n\right) M$$

Matrix Geometric Mean

The minimal geodesic $\gamma^* : [0,1] \mapsto \mathbf{S}_n^+$ connecting $\gamma(0) = S_d$ and $\gamma(1) = S_1^{-1}$, associated with the Riemannian metric $g_A(S_d, S_1^{-1}) = \text{tr}\left(A^{-1} S_d A^{-1} S_1^{-1}\right)$, is

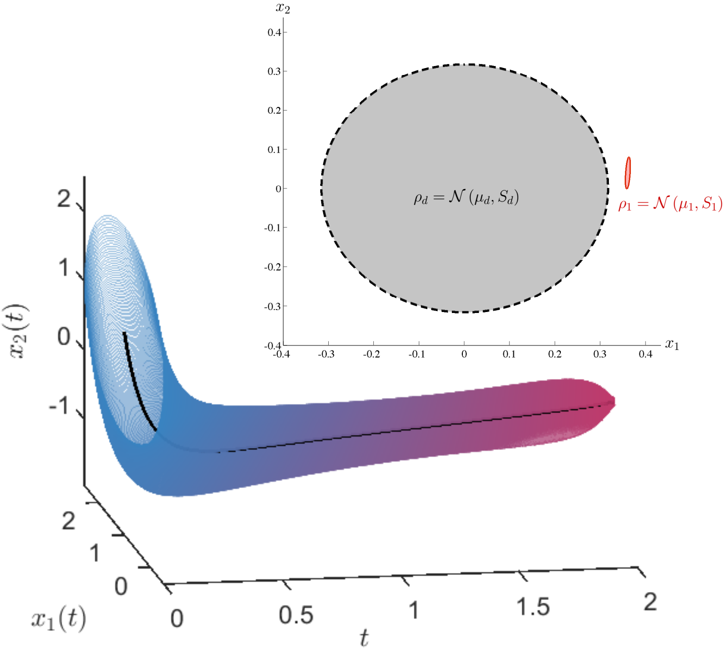
$$\gamma^*(t) = S_d \,\#_t\, S_1^{-1} = S_d^{\frac{1}{2}} \left(S_d^{-\frac{1}{2}} S_1^{-1} S_d^{-\frac{1}{2}} \right)^t S_d^{\frac{1}{2}}$$

$$= S_1^{-1} \,\#_{1-t}\, S_d = S_1^{-\frac{1}{2}} \left(S_1^{\frac{1}{2}} S_d S_1^{\frac{1}{2}} \right)^{1-t} S_1^{-\frac{1}{2}}$$

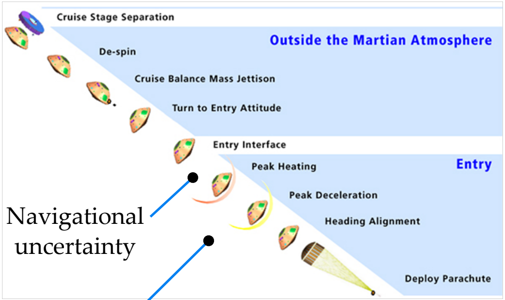
Geometric Mean:

$$\gamma^*\left(\frac{1}{2}\right) = S_d \,\#_{\frac{1}{2}}\, S_1^{-1} = S_1^{-1} \,\#_{\frac{1}{2}}\, S_d$$

Controlled State Covariance



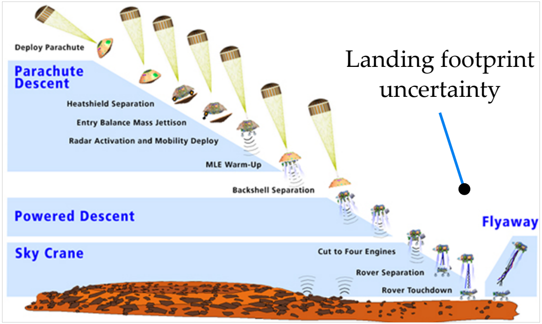
Application: Control for Mars Landing



Navigational uncertainty

Heating uncertainty

Chute deployment uncertainty



Landing footprint uncertainty

Image credit: NASA JPL

Part III. Ongoing and Future Research

UTM

Unmanned Aerial Systems Traffic Management

Vision for UAS Traffic Management (UTM)

Class G airspace extends up to 1200 ft AGL

500 ft AGL

Weight no more than 55 lbs

200 ft AGL

Requires: Automated V2V separation management

Yield manned traffic

Avoid obstacles (buildings, towers etc.)

Technical Challenges

Dynamic Geofencing

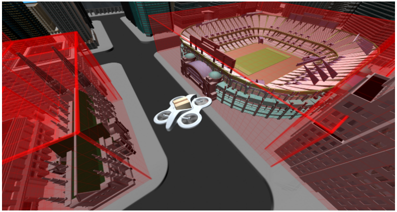
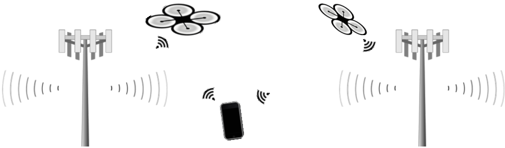
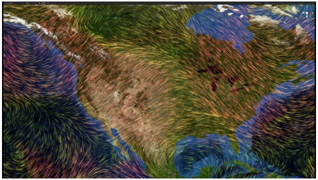


Image credit: NASA Ames Research Center

Control over LTE



Wind Uncertainty



Provable Safety

Protocols $\equiv$ Laws of the Sky

Offline Protocol

– How FAA approves a flight path request?

Motion Protocol

– What does an individual drone do in real time?

Communication Protocol

– What and how should a drone in flight talk?

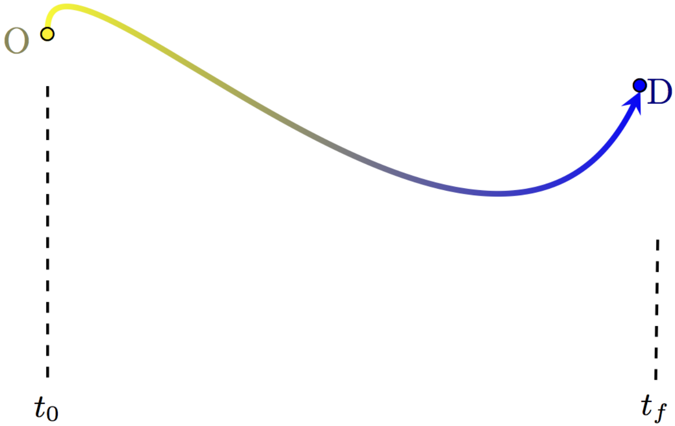
Database Protocol

– Which other drones to talk with and when?

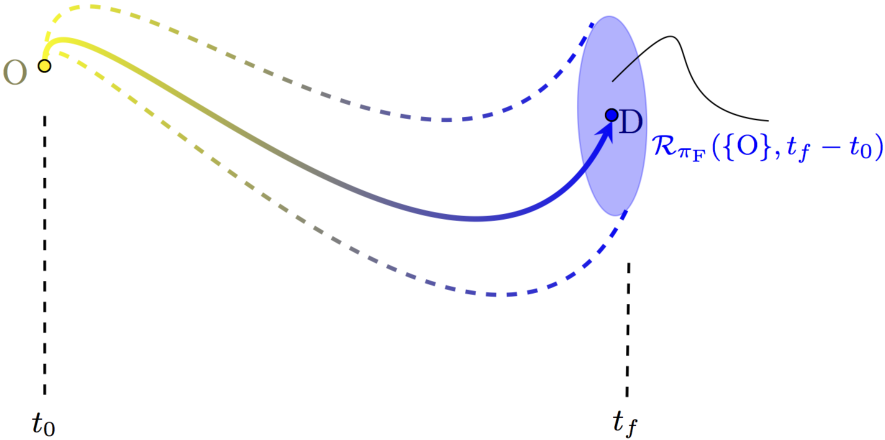
Motion Protocol

What does an individual drone do in real time?

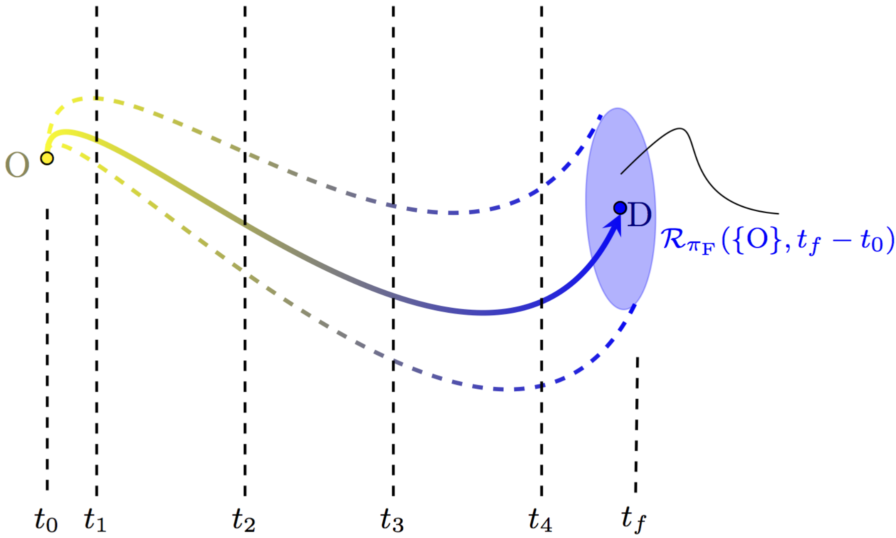
Input: Approved Flight Path



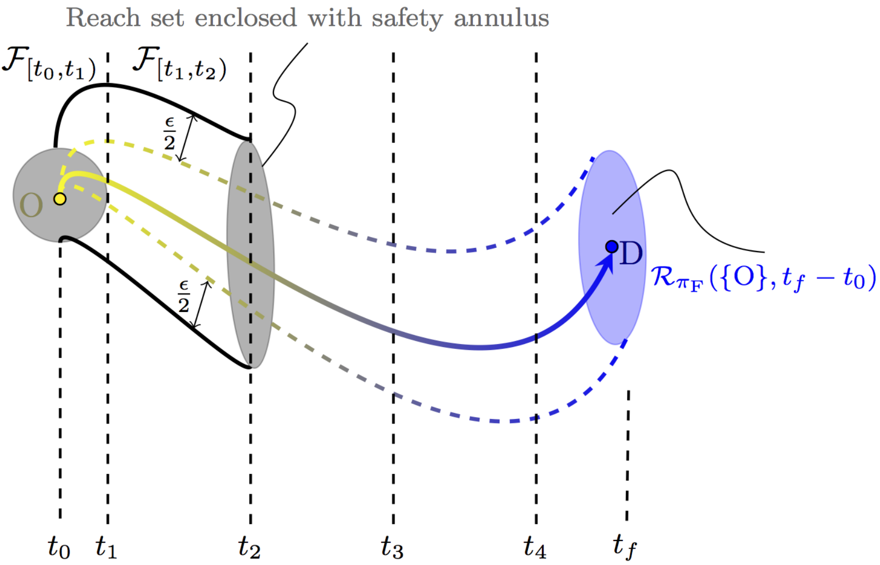
Reach Set Evolution due to Wind Uncertainty



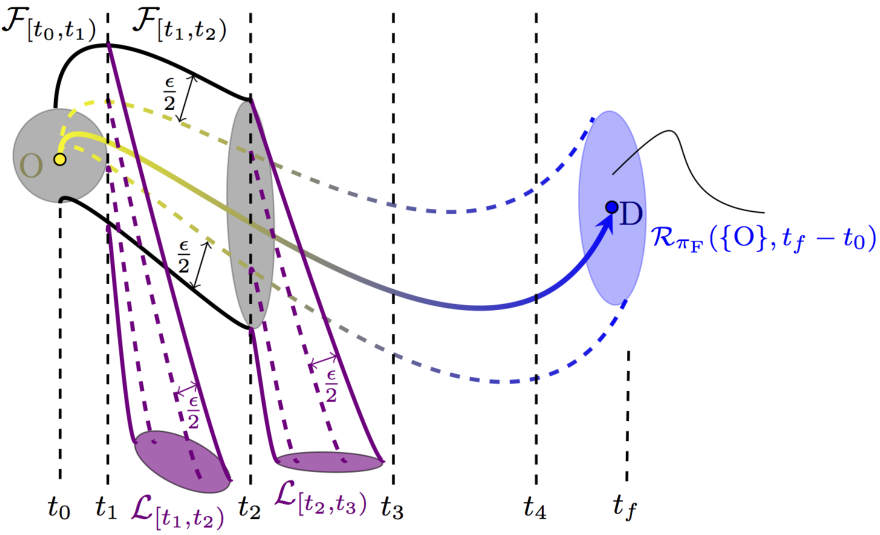
Discrete Decision Making Instances



4D Flight Tubes $\mathcal{F}_{[t_j,t_{j+1})}$

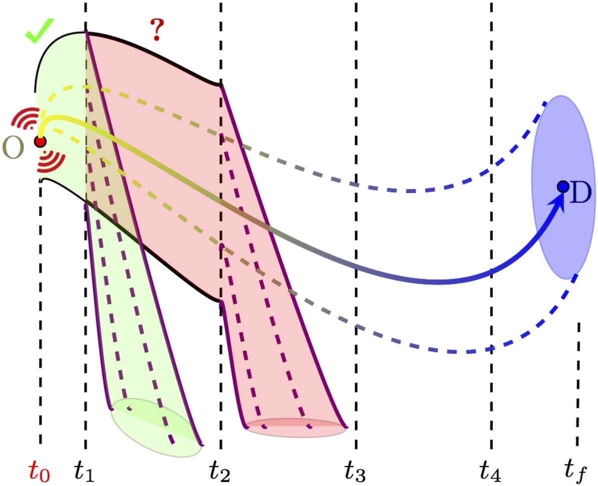


4D Flight + Landing Tubes $\{\mathcal{F}_{[t_j,t_{j+1})}, \mathcal{L}_{[t_{j+1},t_{j+2})}\}$



Motion Protocol: $t = t_0$

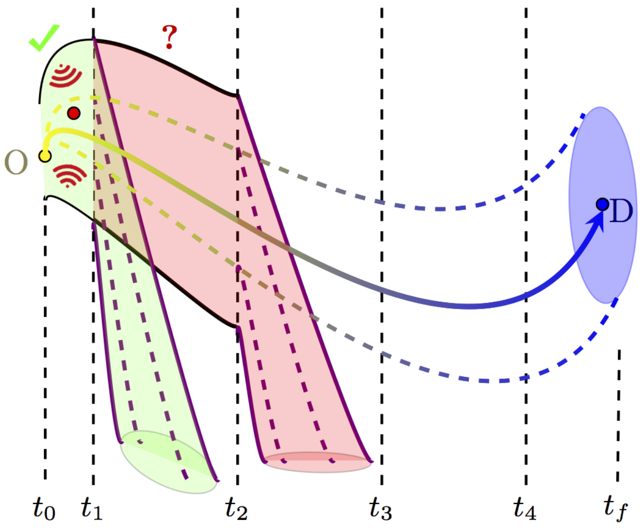
IF: Have all + ACKs for $\{\mathcal{F}_{[t_0,t_1)}, \mathcal{L}_{[t_1,t_2)}\}$ **AND** D $\in \mathcal{R}_{\pi_F}\left(\{O\}, t_f - t_0\right)$



THEN: Take-off **AND** broadcast req. for $\{\mathcal{F}_{[t_1,t_2)}, \mathcal{L}_{[t_2,t_3)}\}$

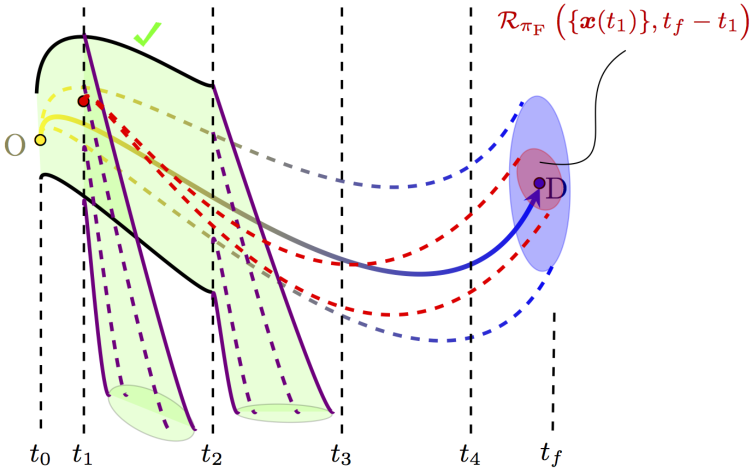
Motion Protocol: $t \in [t_0, t_1)$

Listening for ± ACKs, $\boldsymbol{x}(t) \in \mathcal{F}_{[t_0,t_1)}$



Motion Protocol: $t = t_1$

IF: All + ACKs **AND** D $\in \mathcal{R}_{\pi_F}\left(\{\boldsymbol{x}(t_1)\}, t_f - t_1\right)$

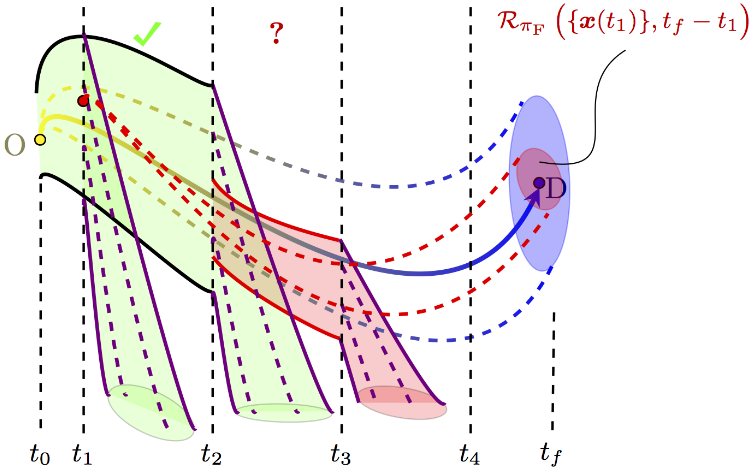


THEN: Continue in $\mathcal{F}_{[t_1,t_2)}$ **AND** broadcast req. for $\{\mathcal{F}_{[t_2,t_3)}, \mathcal{L}_{[t_3,t_4)}\}$

ELSE: Abort mission via $\mathcal{L}_{[t_1,t_2)}$

Motion Protocol: $t = t_1$

IF: All + ACKs **AND** D $\in \mathcal{R}_{\pi_F}\left(\{\boldsymbol{x}(t_1)\}, t_f - t_1\right)$

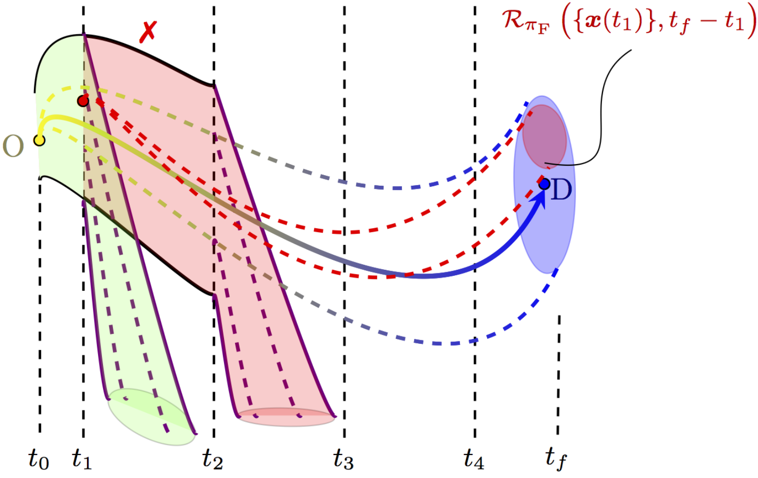


THEN: Continue in $\mathcal{F}_{[t_1,t_2)}$ **AND** broadcast req. for $\{\mathcal{F}_{[t_2,t_3)}, \mathcal{L}_{[t_3,t_4)}\}$

ELSE: Abort mission via $\mathcal{L}_{[t_1,t_2)}$

Motion Protocol: $t = t_1$

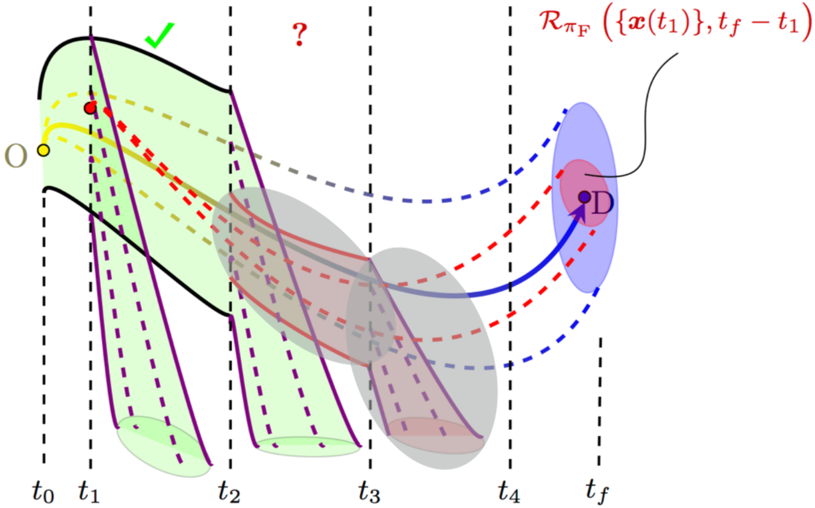
IF: All + ACKs **AND** D $\notin \mathcal{R}_{\pi_F}\left(\{\boldsymbol{x}(t_1)\}, t_f - t_1\right)$



THEN: Continue in $\mathcal{F}_{[t_1,t_2)}$ **AND** broadcast req. for $\{\mathcal{F}_{[t_2,t_3)}, \mathcal{L}_{[t_3,t_4)}\}$

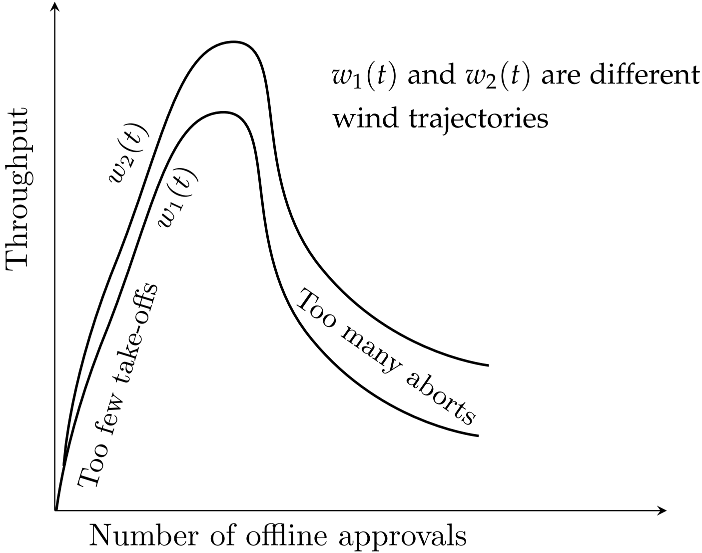
ELSE: Abort mission via $\mathcal{L}_{[t_1,t_2)}$

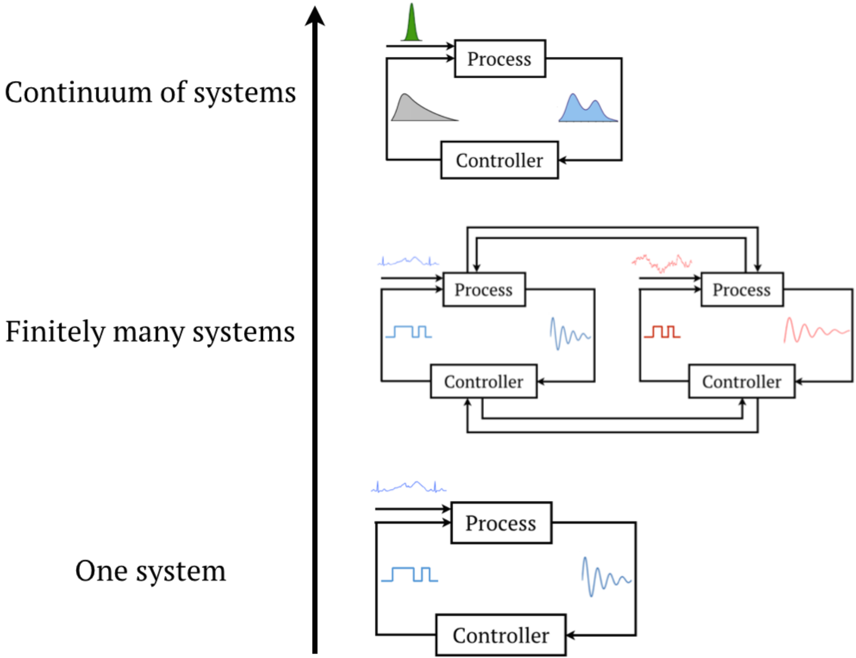
Algorithms for Motion Protocol



Compute minimum volume outer ellipsoids: SDP

Proposed Architecture: Performance

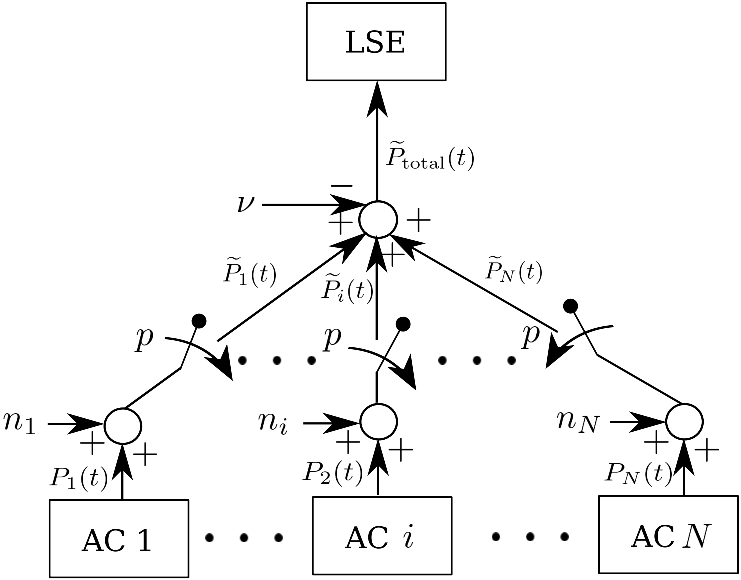



Continuum of systems
Finitely many systems
One system
Process
Controller
Process
Controller
Process
Controller
Process
Controller

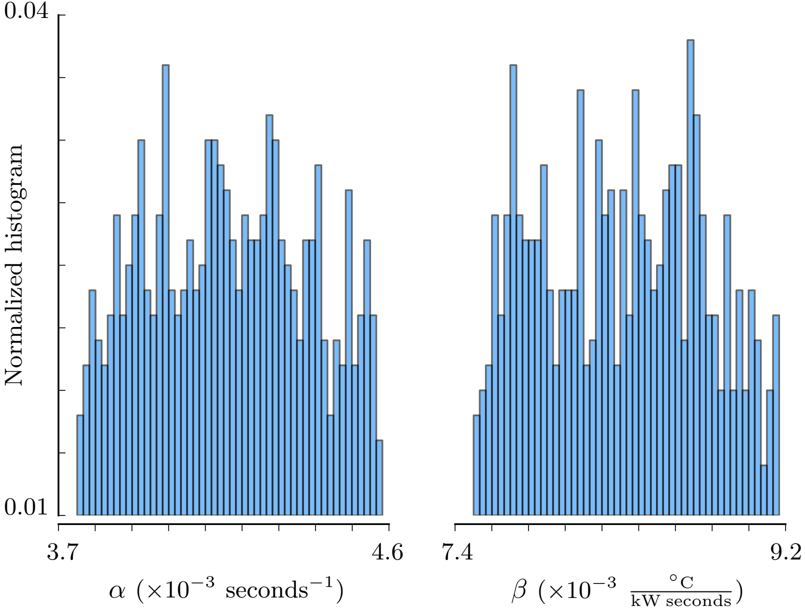
Thank You

Backup Slides for Part I

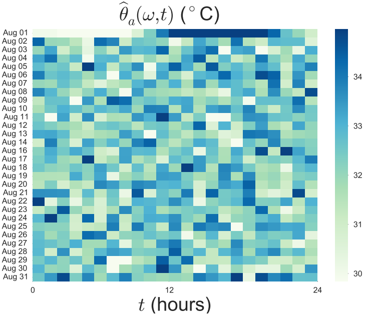
Differential Privacy Preserving Sensing

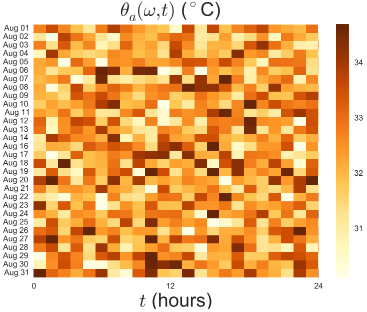


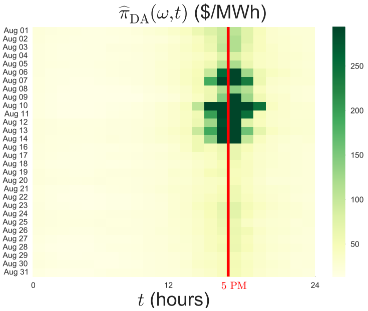
Distribution of Parameters α and β

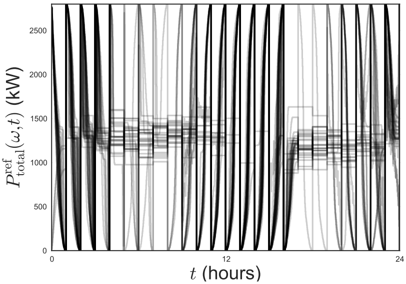


Houston Data for August 2015

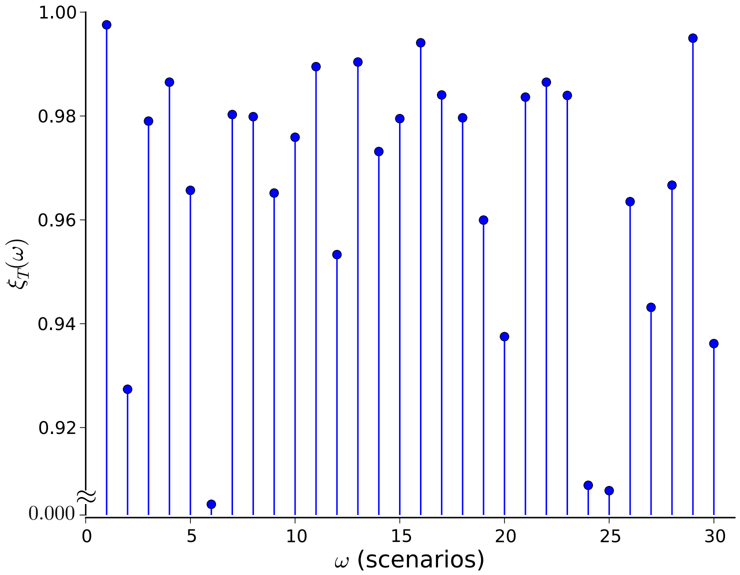








Limits of Control Performance



Backup Slides for Part II

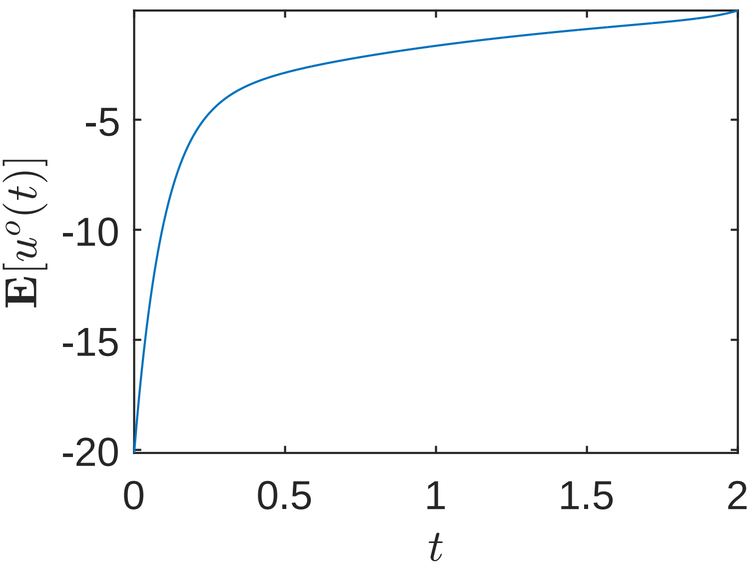
Example

$$\begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mathrm{d}t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \, \mathrm{d}t + \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix} \mathrm{d}w$$

$$\rho_0 = \mathcal{N}\left((1,1)^\top, I_2\right), \quad \rho_d = \mathcal{N}\left((0,0)^\top, 0.1\, I_2\right),$$

$$Q = 100\, I_2, \quad R = 1, \quad M = I_2, \quad t_1 = 2$$

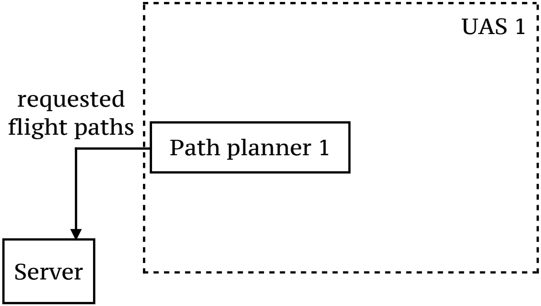
Expected Optimal Control



Backup Slides for Part III

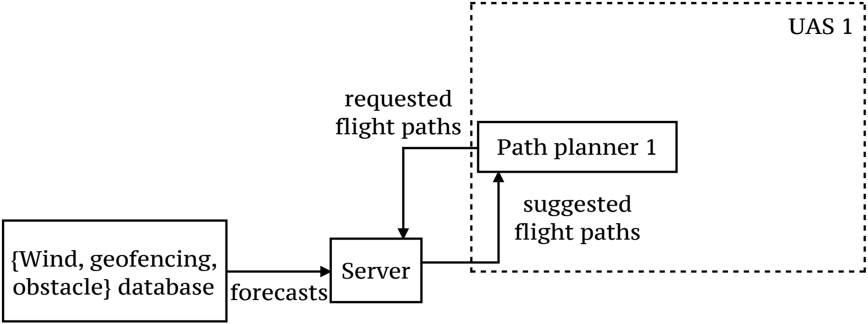
Offline Protocol

Path Planning and Deconfliction



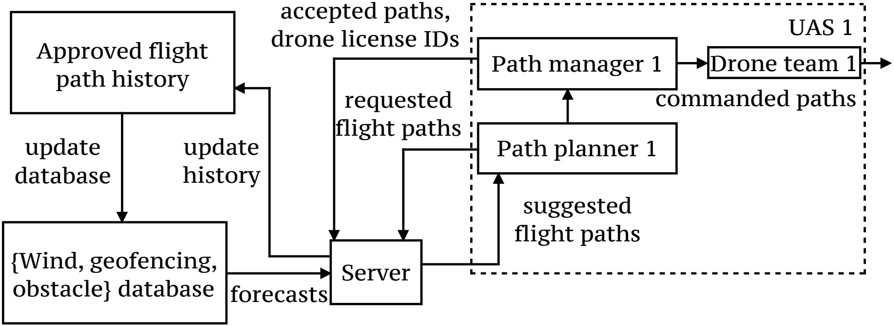
Offline Protocol

Path Planning and Deconfliction



Offline Protocol

Path Planning and Deconfliction



Offline Protocol

Path Planning and Deconfliction