A Control Framework for Demand Response of Thermal Inertial Loads

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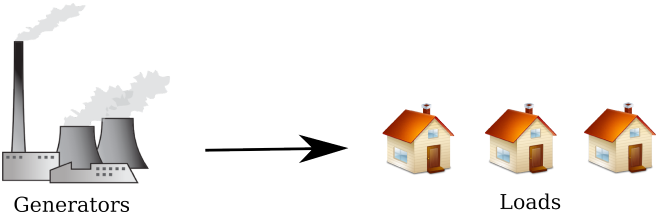
Joint work with X. Geng, G. Sharma, L. Xie, and P.R. Kumar

Demand Response: what, why, how

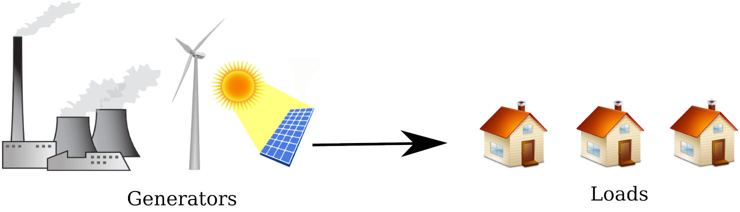


Traditional paradigm: demand is uncertain

Operational model: supply follows demand

Mechanism: operating reserve

Demand Response: what, why, how



New paradigm: both supply and demand are uncertain

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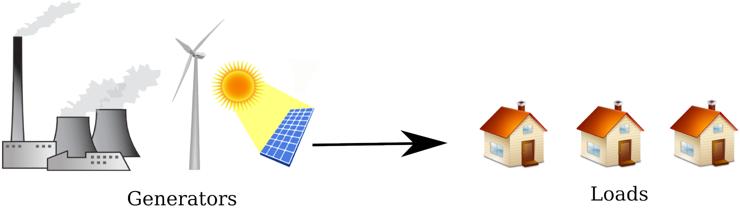
Demand Response: what, why, how

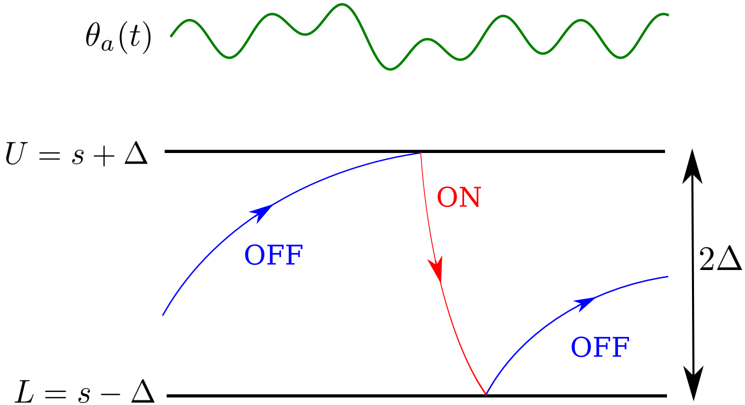


New paradigm: both supply and demand are uncertain

Operational model: demand follows supply

Mechanism: demand response **of thermal inertial loads**

Dynamics of AC state $(s, \theta, \sigma) \in \mathbb{R}^2 \times \{0, 1\}$



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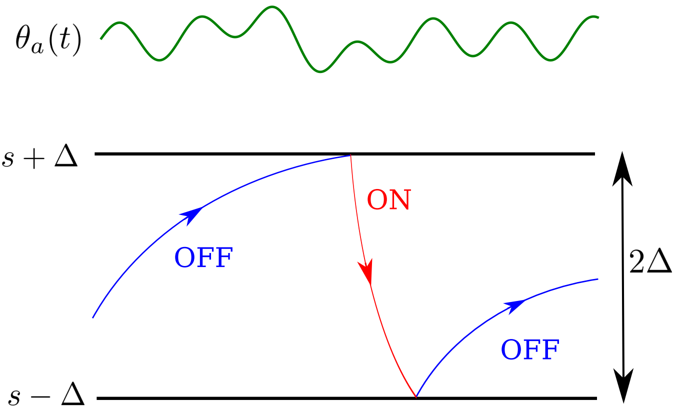
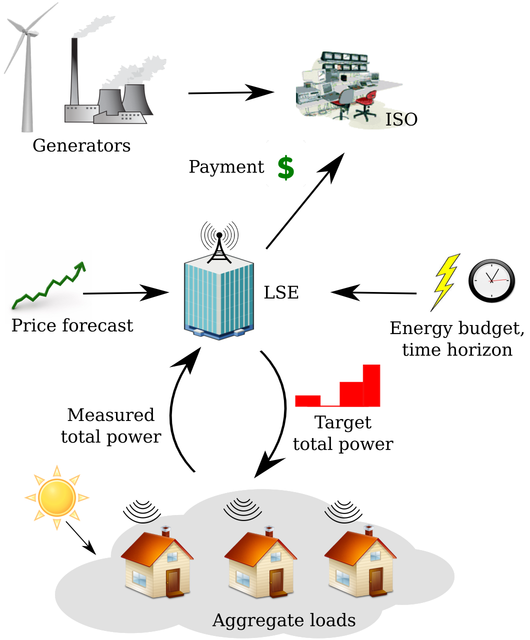


Newton's law of heating/cooling: $\dot{\theta} = -\alpha\,(\theta(t) - \theta_a(t)) - \beta P \sigma(t)$

ON/OFF mode switching: $\sigma(t) = \begin{cases} 1 & \text{if} \quad \theta(t) \geq U \\ 0 & \text{if} \quad \theta(t) \leq L \\ \sigma(t^-) & \text{otherwise} \end{cases}$

Proposed architecture



Research scope

Objective: A theory of operation for the LSE

Challenges:

1. How to design the **target consumption as a function of price**?

2. How to control so as to preserve **privacy** of the loads' states?

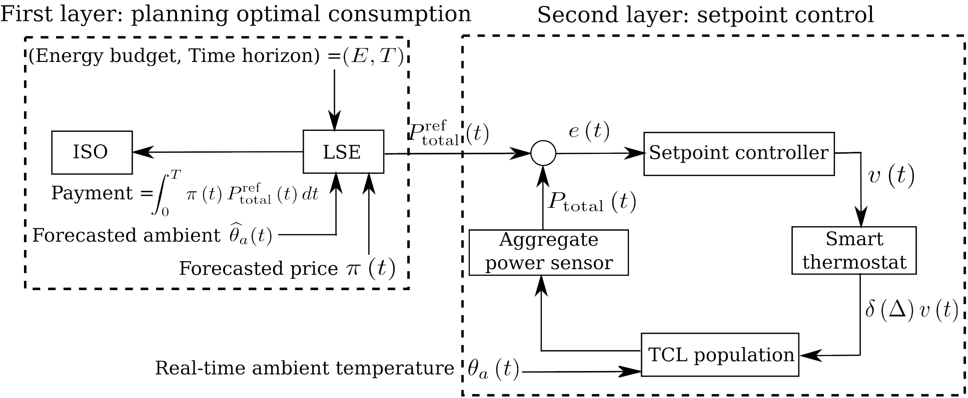
3. How to respect loads' **contractual obligations** (e.g. comfort range width Δ)?

Problem types

Price type / Load type	**Day ahead**	**Real time**
Single large commercial	...	...
Many homes	...	...

Let's focus on **many homes + day ahead price**

Two layer block diagram



First layer: planning optimal consumption

price forecast

$$\underset{\{u_1(t),\ldots,u_N(t)\}\in\{0,1\}^N}{\text{minimize}} \quad \int_0^T P \quad \pi(t) \quad (u_1(t)+u_2(t)+\ldots+u_N(t))\ \mathrm{d}t$$

subject to

$$(1) \quad \dot{\theta}_i = -\alpha\left(\theta_i(t) - \widehat{\theta}_a(t)\right) - \beta P u_i(t) \qquad \forall\, i = 1,\ldots,N,$$

$$(2) \quad \int_0^T (u_1(t)+u_2(t)+\ldots+u_N(t))\ \mathrm{d}t = \tau \doteq \frac{E}{P}(< T, \text{given})$$

$$(3) \quad L_0^{(i)} \le \theta_i(t) \le U_0^{(i)} \qquad \forall\, i = 1,\ldots,N.$$

Optimal consumption: $P^*_{\text{ref}}(t) = P\sum_{i=1}^{N} u_i^*(t)$

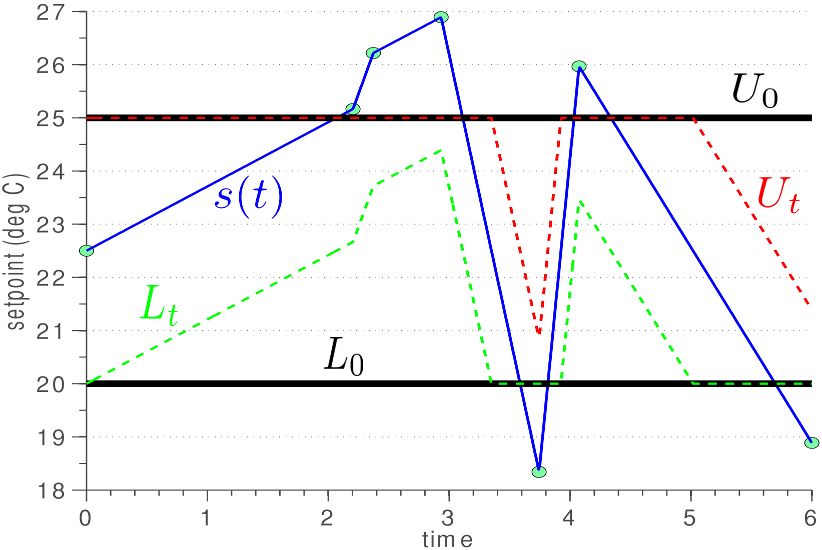
Second layer: setpoint control

$$\underbrace{P^*_{\text{ref}}(t)}_{\text{optimal reference}} = P\sum_{i=1}^{N} u_i^*(t), \quad \rightsquigarrow \quad \underbrace{e(t)}_{\text{error}} = P^*_{\text{ref}}(t) - \underbrace{P(t)}_{\text{measured}}, \quad \rightsquigarrow$$

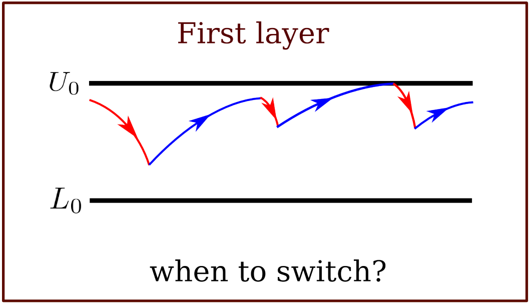
$$v(t) = \underbrace{k_p e(t) + k_i \int_0^t e(\varsigma)\,\mathrm{d}\varsigma + k_i \frac{\mathrm{d}}{\mathrm{d}t} e(t)}_{\text{PID velocity control}}, \rightsquigarrow \frac{\mathrm{d}s_i}{\mathrm{d}t} = \underbrace{\Delta_i}_{\text{gain}} \underbrace{v(t)}_{\text{broadcast}},$$

$$\rightsquigarrow \quad L_t^{(i)} = L_0^{(i)} \vee (s_i(t) - \Delta_i), \qquad U_t^{(i)} = U_0^{(i)} \wedge (s_i(t) + \Delta_i).$$

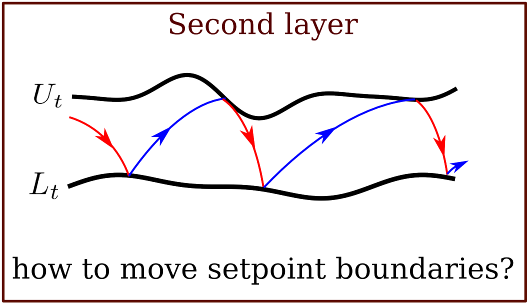
Second layer: setpoint control



Control problems



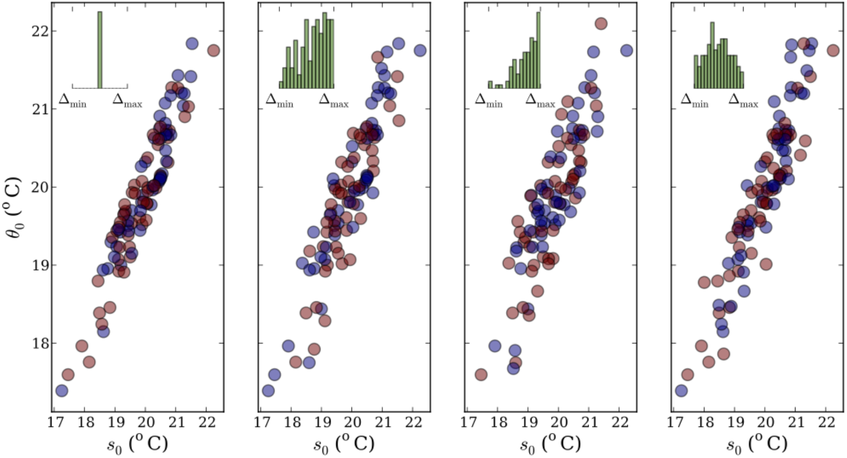
control variable
$\sigma(t)$



control variable
$\frac{\mathrm{d}s}{\mathrm{d}t}$

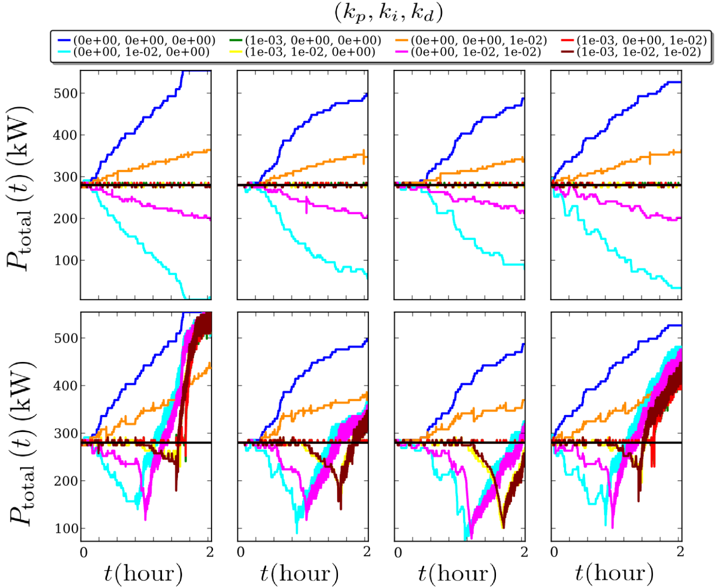
Direct numerical solution

Given: distribution of the $N = 100$ loads' initial conditions $(s_0, \theta_0, \sigma_0)$, and their contracts (Δ)

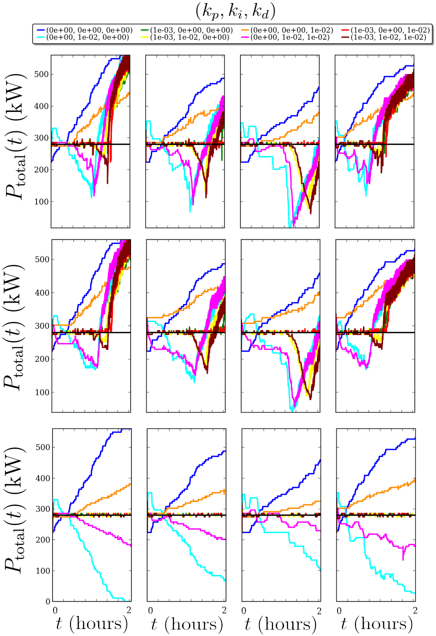


Direct numerical solution: $P^*_{\mathbf{ref}}(t) = 50P$

Setpoint velocity control has good tracking performance

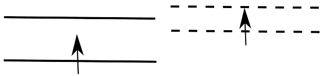


Fairness in setpoint velocity control

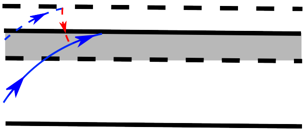


What does "fairness" mean?

all deadbands hit zero at the same time

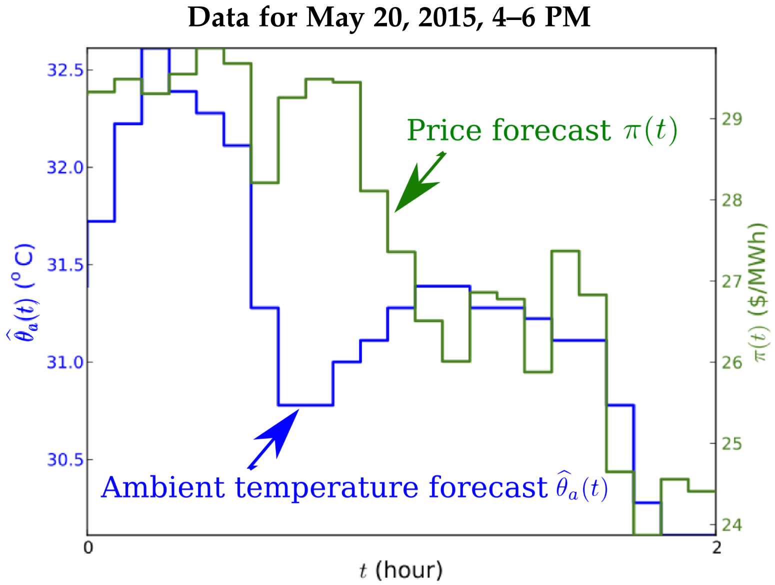


identical states (room temperatures)
see identical controls (setpoint velocity)



no contractual constraints, fairness is not an issue

Direct numerical solution: Houston data



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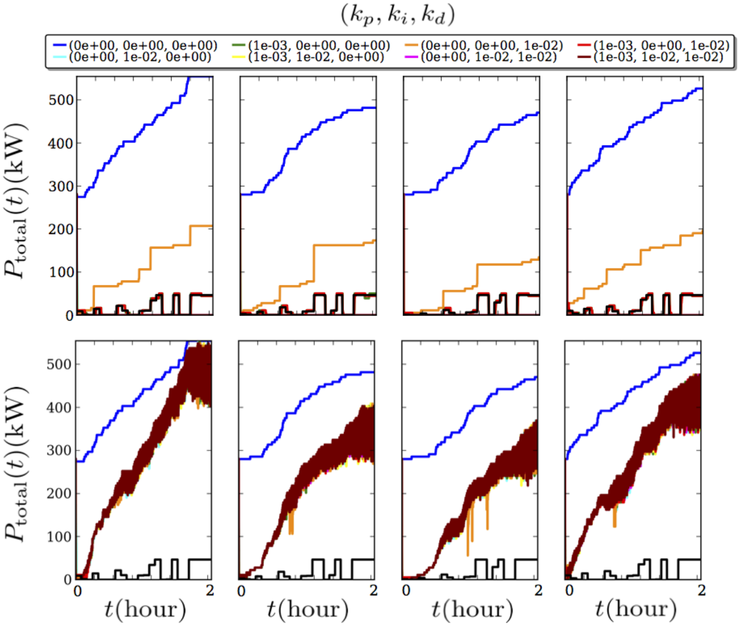

Analytical solution for planning problem

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Intuition: what if price were monotone in time?

Assume: $N = 1$ **home. Constraints (1) and (2) active.**

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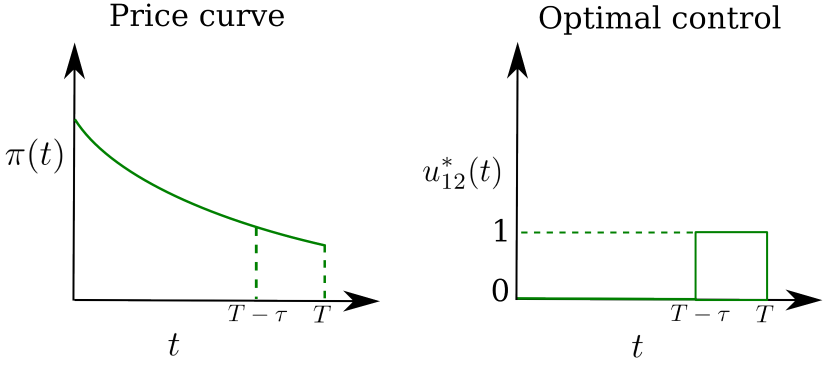

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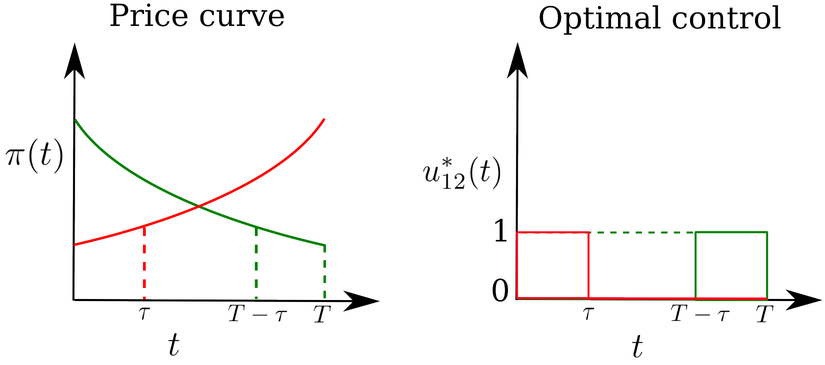
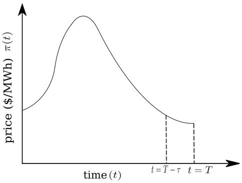


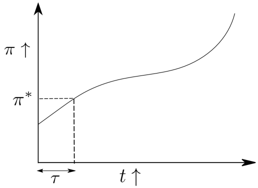
Analytical solution for planning problem

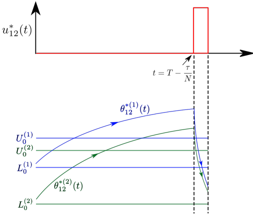
$N \geq 1$ **homes. Constraints (1) and (2) active.**

$$F_\pi(\widetilde{\pi}) \triangleq \int_0^T \mathbf{1}_{\{\pi(t) \leq \widetilde{\pi}\}}\, dt, \quad \pi^* \triangleq \inf\{\widetilde{\pi} \in \mathbb{R}^+ : F_\pi(\widetilde{\pi}) = \tau\},$$

$$S \triangleq \{s \in [0, T] : \pi(s) < \pi^*\}, \quad u^*(t) = \begin{cases} 1 & \forall t \in S, \\ 0 & \text{otherwise.} \end{cases}$$







Optimal actions are synchronized

Analytical solution for planning problem

Constraints (1), (2) and (3) active.

Case I: large $\Delta \Leftrightarrow \exists \Theta_0$ **s.t.** $\forall \theta_0 \in \Theta_0, \theta^*_{123}(t) = \theta^*_{12}(t)$

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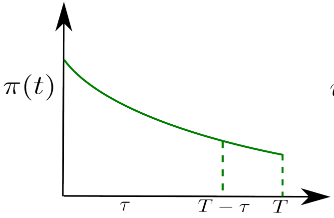

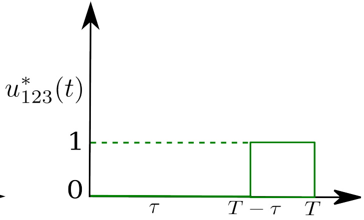

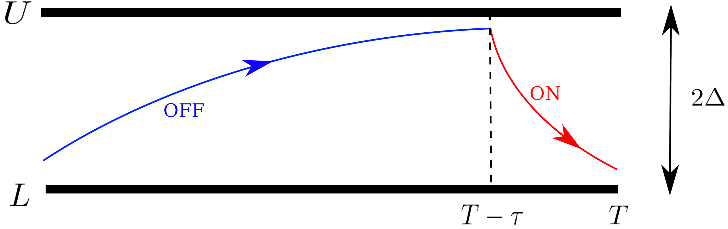

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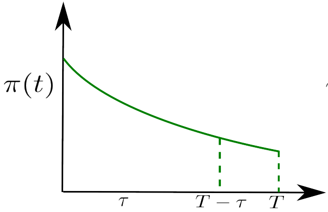

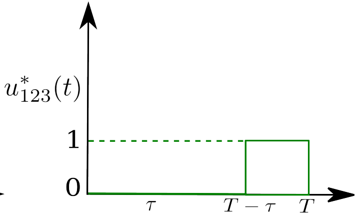

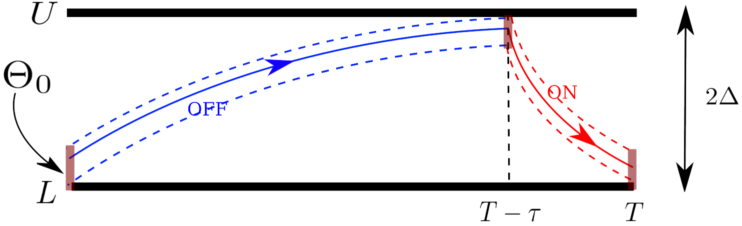

Understanding large Δ condition (lin traj)

Suppose $\dot{\theta} = \begin{cases} +\alpha \\ -\beta \end{cases}$. We have

$$\begin{aligned} 2\Delta &> \alpha\,(T-\tau) \vee \beta\tau \\ &\Updownarrow \\ \exists\ \Theta_0 &\doteq \underbrace{\left[L + \left[(\alpha+\beta)\,\tau - \alpha T\right]^{+}\right.}_{\begin{cases} = L \text{ for } \frac{\tau}{T} \in (0, \frac{\alpha}{\alpha+\beta}] \\ > L \text{ for } \frac{\tau}{T} \in (\frac{\alpha}{\alpha+\beta}, 1] \end{cases}},\ \underbrace{U - \alpha\,(T-\tau)\big]}_{L \leq \quad < U} \end{aligned}$$

If $\theta_0 \in \Theta_0$, then optimal policy = $\begin{cases} \text{OFF} & \forall\, t \in (0, T-\tau) \\ \text{ON} & \forall\, t \in [T-\tau, T] \end{cases}$

i.e., $\theta^*_{123}(t) = \theta^*_{12}(t)$

Understanding large Δ condition (exp traj)

Suppose $\dot{\theta} = -\alpha\left(\theta(t) - \theta_a\right) - \beta P u$. We have

$$\begin{aligned} 2\Delta &> \left(L\left(e^{\alpha\tau} - 1\right) + \theta_a + \frac{\beta}{\alpha}P\right) \vee \left(\left(\theta_a - U\right)\left(e^{\alpha(T-\tau)} - 1\right)\right) \\ &\Updownarrow \\ \exists\, \Theta_0 &\doteq \left[L \vee \left(\theta_a + e^{\alpha T}\left(L - 2\theta_a e^{-\alpha\tau} + \frac{\beta}{\alpha}P e^{-\alpha\tau}\right)\right), \underbrace{\left(U - \theta_a\right)e^{\alpha(T-\tau)} + \theta_a}_{L \leq \quad < U}\right] \end{aligned}$$

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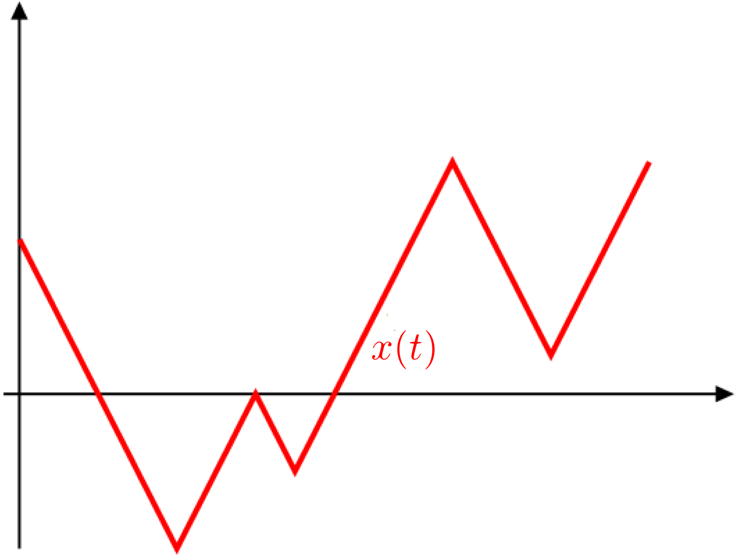
i.e., $\theta^*_{123}(t) = \theta^*_{12}(t)$

Analytical solution for planning problem

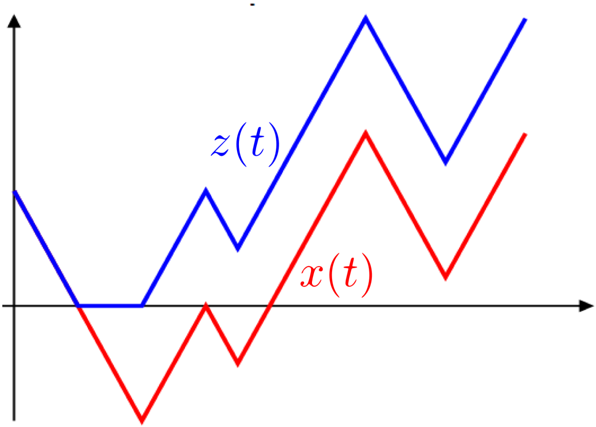
Constraints (1), (2) and (3) active.

Case II: $\theta_{123}^{*(i)}(t) = \Psi_{L_0^{(i)},U_0^{(i)}}\left(\theta_{12}^{*(i)}(t)\right)$**, where** $\Psi_{L,U}(\cdot)$ **is the two-sided Skorokhod map in** $[L, U]$

Digression: Skorokhod map Ψ

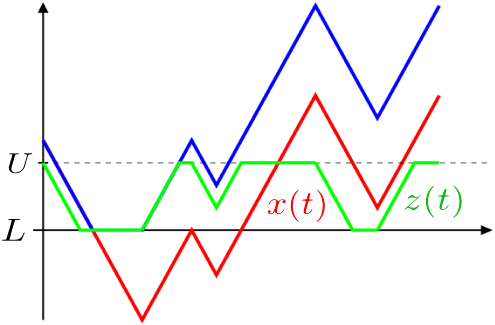


Digression: Skorokhod map $\Psi_{0,\infty}$



$$z(t) = x(t) + \sup_{0 \le s \le t} [-x(s)]^+ \quad \text{(Skorokhod, 1961)}$$

Digression: Two-sided Skorokhod map $\Psi_{L,U}$



$$z(t) = \Lambda_{L,U} \circ \Psi_{L,\infty}(x(t))$$

$$\Lambda_{L,U}(\phi(t)) = \phi(t) - \sup_{0\le s\le t}\left([\phi(s) - U]^+ \wedge \inf_{s\le r\le t}(\phi(r) - L)\right)$$

$$\Psi_{L,\infty}(x(t)) = x(t) + \sup_{0\le s\le t}[L - x(s)]^+$$

(Kruk, Lehoczky, Ramanan, Shreve, 2007)

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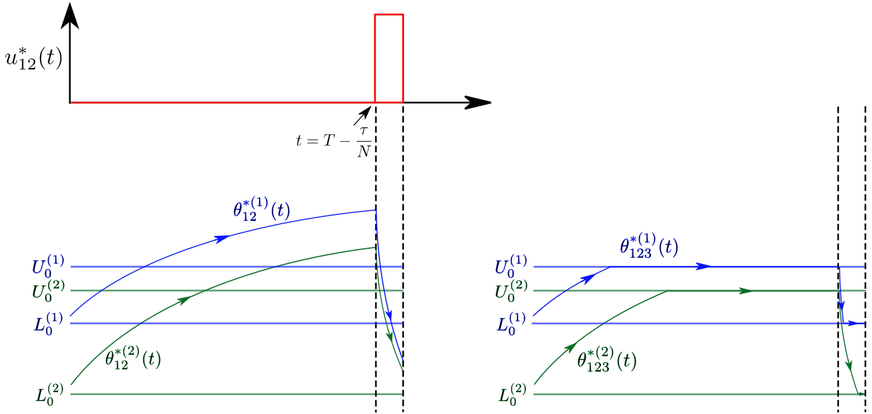

Summary

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- Designs optimal target consumption using forecast.
- Tracks the designed target consumption in real-time.
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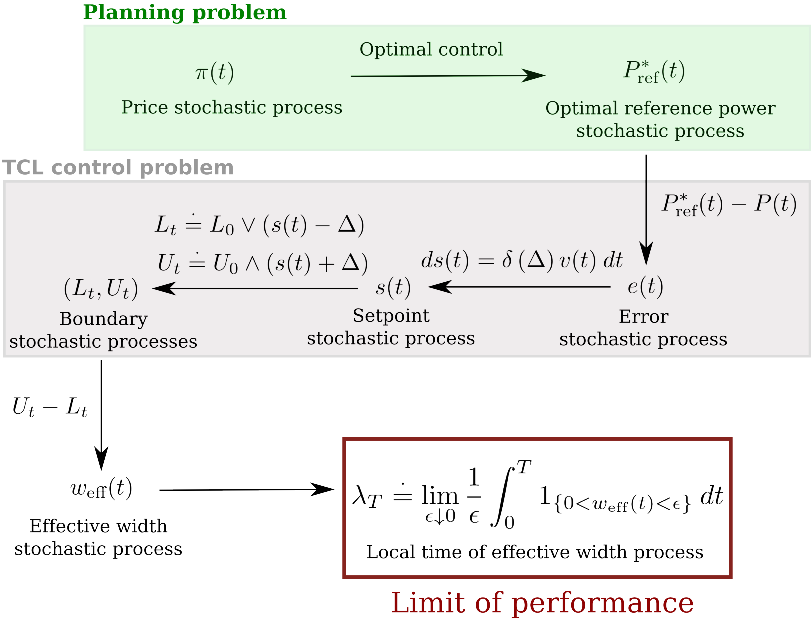
Thank you

Performance



Real time market + large commercial load

$$\underset{u(\cdot)\,\in\,\mathbf{1}_{\mathcal{P}\left(\theta,\pi_{\mathrm{RT}},\theta_a\right)}}{\text{minimize}} \quad \mathbb{E}\left[\int_0^T \left\{\pi_{\mathrm{RT}} P u \;+\; \gamma\left(\theta-\theta_d\right)^2\right\} dt\right]$$

subject to

(1) $\dot{\theta}(t) = -\alpha\left(\theta(t) - \theta_a(t)\right) - \beta P u(t),$

[ODE for continuous state θ]

(2) $\mathbf{m} \triangleq (\pi_{\mathrm{RT}}, \theta_a) \sim Q = Q_{\pi_{\mathrm{RT}}} \otimes Q_{\theta_a}.$

[finite state continuous time Markov chain for $\mathbf{m}$]

State: $(\theta, \mathbf{m}) \in \mathbb{R} \times |\mathcal{M}|$, where $|\mathcal{M}| = n_{\pi_{\mathrm{RT}}} n_{\theta_a}$

Find: optimal (indicator) feedback $u^*(t) = \mathbf{1}_{\mathcal{P}(\theta,\mathbf{m})} \in \{0, 1\}$

HJB for controlled Markov jump process

Value function: $V_i \triangleq V(\theta, \mathbf{m} = i),\ i = 1, 2, \ldots, |\mathcal{M}|$

HJB:

$$0 = \inf_{u(\cdot) \in \mathbf{1}_{\mathcal{P}(\theta, \pi_{\text{RT}}, \theta_a)}} \left[\pi_{\text{RT}} P u + \gamma (\theta - \theta_d)^2 + \frac{\partial V_i}{\partial t} \right.$$

$$\left. + \frac{\partial V_i}{\partial \theta} \left\{ -\alpha (\theta - \theta_a) - \beta P u \right\} + \sum_{j=1}^{|\mathcal{M}|} q_{ij} \left(V_j - V_i \right) \right]$$

$\forall i = 1, 2, \ldots, |\mathcal{M}|$

Involves optimization problem:

$$\inf_{u(\cdot)} \underbrace{\left[\pi_{\text{RT}} P u + \frac{\partial V_i}{\partial \theta} \left\{ -\alpha (\theta - \theta_a) - \beta P u \right\} \right]}_{\Gamma(u)}$$

$\Rightarrow$ If $\Gamma(1) - \Gamma(0) = \pi_{\text{RT}} P - \beta P \frac{\partial V_i}{\partial \theta} < (>) 0$, then $u^* = 1(0)$

What can we tell about the value function

Optimality condition: If $\frac{\partial V_i}{\partial \theta} > (<) \frac{\pi_{\text{RT}}(t)}{\beta}$, then $u^*(t) = 1(0)$

Notice:
Optimality condition is invariant under convexification
$u \in \{0, 1\} \mapsto u \in [0, 1]$

Lemma: $V_{i_{[0,1]}}$ is convex in θ.

Ongoing: code for value iteration, Q-learning.

Value iteration

Bellman equation:

$$V_k(i) = \min_{u \in \{0,1\}} \left[c_k\left(x = i, u\right) + \sum_{j \in \mathcal{X}} p_{ij}\left(u\right) V_{k+1}\left(j\right) \right], \ V_T = \text{zeros}(n, 1).$$

Suppose we make 100 discretizations for $\theta \in [18, 22]$, and 40 discretizations for price $\pi_{\text{RT}} \in [50, 100]$. Let's make ambient $\theta_a = 32$ deg Celcius (constant). Then state space is a 100×40 grid. In Bellman equation, $n = 100 \times 40 = 4000$, and the indices $i, j = 1, 2, \ldots, n$. The time index k runs backwards. So $k + 1 \mapsto k$ means a negative 15 minutes time-step. Take actual final time $T = 2 * 3600$. $[p_{ij}]$ is a transition probability matrix of size $n \times n = 4000 \times 4000$, and is constructed as $P = P_\theta \otimes P_{\pi_{\text{RT}}}$, where P_θ is of size 100×100, and $P_{\pi_{\text{RT}}}$ is of size 40×40. The symbol $\otimes$ denotes kronecker product (MATLAB `kron`).