Aero 320: Numerical Methods

Homework 1

Please submit your code (.cpp and .m files), figure (.pdf file) and write up (for part (b) and (c))

Due: September 11, 2013

Problem 1

Truncation and round-off error: derivative approximation revisited

Consider the function $f(x) = \exp(100x)$ as in Lab Assignment 2.

(a) Write a program to compute the *approximate* value of $f'(x)$ evaluated at $x = 0$ using the following approximation of the derivative

$$f'(x) \approx \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

for different choices of $h = 2^{-k/4}$, where k varies from 20 to 200. Your code should output h and relative error as two columns in a file.

(b) Load your data file in MATLAB and plot the relative error (in the vertical axis) versus h (in horizontal axis) in log-log scale. Do this plot for both single precision and double precision arithmetic, on the same MATLAB figure. What are your conclusions from this plot?

(c) On the same figure generated in part (b), plot the results of Lab 2. Label your plots so that you know which curve is what. Looking at all the 4 plots on the same figure, explain the difference in results in part (b) and Lab 2.

(d) Run the following commands from MATLAB command line, to save your MATLAB figure as a high quality .pdf file. Before running these commands, please make sure to keep the figure window open, and you may resize the window manually to adjust the plot area in the .pdf file.

```
>> set(gcf, 'PaperPositionMode', 'auto')
>> print -dpdf 'YourFilename.pdf'
```

Solution to Problem 1

(a) See attached `hw1.cpp` file.

(b) See attached `HW1_ImportData_Plot.m` file. This plot shows that the double precision computation (*magenta, dashed line with squares*) results smaller *total error* compared to the single precision computation (*green, solid line with squares*). For smaller values of h, the round-off errors dominate (the respective curves with *negative* slope) and hence the double precision (*magenta*) negative slope segment stays much lower than the single precision (*green*) negative slope segment. This makes sense because the variable type `double` causes less round-off error than `float`. However, when the *truncation errors* start to dominate (the respective curves with *positive* slope), then both the *green* and *magenta* errors increase as h increases. This is intuitive since larger h causes larger *truncation error*. Notice that for large h (when the round-off errors are less significant), the positive slope segments of both the *magenta* and *green* curves, have same slope. This is because these two curves are obtained using the same approximation (called *central difference*) of the derivative and both have the truncation error $O\left(h^2\right)$. Before plotting, you can predict the value of this slope using the truncated Taylor series.

The optimal h occurs when the truncation and round-off errors are approximately same, and the total error is minimum. For this *central difference* approximation, the optimal step-sizes are $h^*_{\text{central float}} \approx 10^{-4}$, and $h^*_{\text{central double}} \approx 10^{-7}$.

(c) We have already compared the *red* and *blue* curves in Lab 2. The conclusions are similar to part (b), the truncation error is $O(h)$, and $h^*_{\text{forward float}} \approx 5 \times 10^{-5}$, and $h^*_{\text{forward double}} \approx 10^{-10}$. Now we want to compare the "blue-red pair" with "green-magenta pair". When round-off errors dominate (negative slope segment), then *green* is slightly below *blue*; and *magenta* is slightly below *red*. This is because the "green-magenta pair" has less *truncation error* than "blue-red pair", although the *blue & green*, and *red & magenta* uses same variable types. The improvement is small since the truncation error itself is small in this range. However, this effect is prominent for larger h (positive slope segment) that shows the difference in slopes due to different truncation errors: $O(h)$ for *forward difference* (Lab 2) approximation, and $O\left(h^2\right)$ for *central difference* approximation (HW1). The conclusion is that higher order Taylor series approximation of the derivative results smaller truncation error, as expected.

(d) See attached `HW1plot.pdf` file.