Aero 320: Numerical Methods

Homework 2

Name: ....

Due: September 24, 2013

NOTE: All problems are to be done by hand (with the help of a calculator) but you need to show all the steps. Turn in a hard copy of your HW stapled with this as cover sheet with your name written in the above field. Submit your HW in the lab next Tuesday. Late submissions or failure to submit in the required format will receive no credit.

Problem 1

More on round-off error

The Taylor series of degree n for $\exp(x)$ is $\sum_{j=1}^{n} \frac{x^j}{j!}$. Use this polynomial and *rounding-off to three digits*, to find an approximate value of $\exp(-5)$ using

(a) $\exp(-5) \approx \sum_{j=1}^{9} \frac{(-5)^j}{j!} = \sum_{j=1}^{9} \frac{(-1)^j \, 5^j}{j!}$,

(b) $\exp(-5) = \frac{1}{\exp(5)} \approx \frac{1}{\sum_{j=1}^{9} \frac{5^j}{j!}}$.

The *exact value* of $\exp(-5)$, correct to three digits, is 6.74×10^{-3}. Which formula (a) or (b) gives more accurate result? Why?

Problem 2

(Problem # 10, p. 67) Convergence of secant versus bisection method

Explain why the secant method (see p. 39) usually converges to a given stopping tolerance faster than bisection.

Problem 3

(Problem # 15, p. 67) Newton's method for finding n^{th} root of a number

Applying Newton's method to the equation $x^2 = N$, gives the following algorithm to compute the square root of N (Problem # 14, p. 67):

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right).$$

(a) Find an algorithm for getting the *cubic* and *quartic* roots of N that have a similar form to the one above for the square root. Can you generalize your answer for the n^{th} root?

(b) Starting with $x_0 = 2$, perform 3 Newton iterations to find $3^{1/3}$. Compare your result with the *actual value* of $3^{1/3}$ (use your calculator), and determine the *number of significant digits* in your answer.

Problem 4

(Problem # 36, p. 69) Fixed point method

Most equations of the form $f(x) = 0$ can be rearranged in the form $x = g(x)$, with which to begin the fixed-point method. For $f(x) = \exp(x) - 2x^2$, one possible way to rewrite $f(x) = 0$ is:

$$x = \pm\sqrt{\frac{\exp(x)}{2}}.$$

(a) Show that this fixed point iteration converges to a root near 1.5 if the positive value is used and to the root near -0.5 if the negative value is used. (Show 3 iterations).

(b) There is a third root near 2.6. Show that we do not converge to this root even though values near to the root are used to begin the iterations. Where does it converge if $x_0 = 2.5$? If $x_0 = 2.7$? (Show 3 iterations).

(c) Find another rearrangement of the form $x = g(x)$, that converges correctly to the third root.

Problem 5

Newton's and Halley's method to solve Kepler's equation

Kepler's equation, devised by Johannes Kepler in 1609, rules the propagation of satellites (or planets) in their orbits. The equation is transcendental:

$$M = E - \epsilon \sin(E)$$

where M is an angle called *mean anomaly*, ϵ is a positive number called *orbit eccentricity*, and E is an angle called *eccentric anomaly* that indicates the location of the satellite in the orbit. Solving Kepler's equation means to find E, when M and ϵ are given.

Solve the Kepler's equation using **Newton's** and **Halley's method** for the following cases. Use $E = M + 1$ as the starting point, and the convergence is achieved if the absolute error $\leq 10^{-6}$. Also report the number of iterations needed to converge for each case.

(a) $\epsilon = 0.2, 0.4, 0.8$, for $M = 0, 1, 10, 100$ degrees.

(b) $\epsilon = 0.9999$ (orbit almost parabolic), for $M = 0.001$ degrees (passage close to perigee).