Aero 320: Numerical Methods

Homework 5

Name: ....

Due: November 4, 2013

NOTE: All problems, unless explicitly asked to write a code, are to be done by hand (with the help of a calculator) but **you need to show all the steps**. Turn in a hard copy of your HW stapled with this as cover sheet with your name written in the above field. Submit your HW by Monday midnight at Room 201, Reed McDonald Building. Late submissions or failure to submit in the required format will receive no credit.

Problem 1

Interpolation using monomial basis and Lagrange polynomials
$(5 + 10 + 10 + 10 + 2 + 3 = 40$ **points)**

Suppose we have observed 4 experimental data points: $(x_0, y_0), \quad (x_1, y_1), \quad (x_2, y_2), \quad (x_3, y_3)$.

(a) If we want to interpolate these data using standard monomial basis, then we need to compute the coefficient vector $c = \{c_0 \quad c_1 \quad c_2 \quad c_3\}^\top$ such that the interpolating polynomial is $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. Find the matrix-vector equation from which vector c can be solved.

(b) From your answer in part (a), under what conditions on the data points (x_i, y_i), $i = 0, \ldots, 3$, we can have unique solution for c?

(c) Let the numerical values of the experimental data are $(0, 0); \quad (0.5, 4.25); \quad (1, 3); \quad (2, 2)$. From your answer in part (b), does unique c exist in this case? If yes, determine the vector c using any algorithm you have learned in this course.

(d) Determine the Lagrange interpolating polynomial that passes through the data points given in part (c).

(e) Compare the Lagrange interpolating polynomial derived in part (d), with the interpolating polynomial having monomial basis derived in part (c). Predict the value of y at $x = 1.5$ from part (c) and part (d).

(f) Submit a plot of the interpolating polynomial(s) in part (e). Mark the data points on your plot. (Hint: If using MATLAB, look up the commands `ezplot` and `plot` in MATLAB Help.

Also, you may find it useful to look at the commands in Homework 1, Problem 1 (d), for saving the MATLAB plot as .pdf file.)

Problem 2

Interpolation using Neville's method and divided differences $(10+10+10 = 30$ **points)**

(a) For the experimental data given in Problem 1(c), construct the Neville table to interpolate the value of y at $x = 1.5$. How does your answer compare with that in Problem 1(e)?

(b) Repeat part (a) with the method of divided differences.

(c) If you are doing polynomial interpolation through a fixed set of $n + 1$ data points, is it possible that different algorithms (monomial basis, Lagrange polynomial, Neville's method, divided difference) can produce different answers? Why/why not?

Problem 3

Least squares $(15 + 15 = 30$ **points)**

Consider the data in the following table.

x_i	1	2	4	5
y_i	2	3	5	6

We want to *approximate* the data using a quadratic polynomial $y = a_0 + a_1 x + a_2 x^2$ in least squares sense.

(a) Set up the matrix-vector equation to solve for the coefficient vector $a = \{a_0 \quad a_1 \quad a_2\}^\top$. Report the polynomial by solving the least squares problem. Submit a plot of the data points together with your least squares approximating polynomial. (Hint: You may use the LU decomposition technique to compute inverse of a square matrix, that you learned in Homework 4, Problem 1(f).)

(b) Approximate the same data using a straight line of the form $y = \alpha_0 + \alpha_1 x$, by resolving the least squares problem. Plot the approximating straight line on top of your plot in part (a). Report the straight line approximation and submit the final plot.