Aero 320: Numerical Methods

Lab Assignment 18

Fall 2013

Problem 1

Numerical integration

Numerically integrate $\int_0^{\frac{\pi}{2}} \cos x \, dx$, using

(a) Midpoint formula with partition $\left[0, \frac{\pi}{4}\right]$, $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

(b) Trapezoid method with the same partition as part (a).

(c) Three point Simpson's method.

Solution

(a) $I_a = \frac{\pi}{4} \times \cos\left(\frac{0+\frac{\pi}{4}}{2}\right) + \frac{\pi}{4} \times \cos\left(\frac{\frac{\pi}{4}+\frac{\pi}{2}}{2}\right) = \frac{\pi}{4}\left(\cos\frac{\pi}{8} + \cos\frac{3\pi}{8}\right) \approx 1.0262.$

(b) $I_b = \frac{1}{2} \times \left(\frac{\pi}{4} - 0\right) \times \left(\cos 0 + \cos\frac{\pi}{4}\right) + \frac{1}{2} \times \left(\frac{\pi}{2} - \frac{\pi}{4}\right) \times \left(\cos\frac{\pi}{4} + \cos\frac{\pi}{2}\right) = \frac{\pi}{8}\left(1+\sqrt{2}\right) \approx 0.9481.$

(b) $I_c = \frac{1}{6}\left(\frac{\pi}{2} - 0\right)\left(\cos 0 + 4\cos\frac{\pi}{4} + \cos\frac{\pi}{2}\right) = \frac{\pi}{12}\left(1 + \frac{4}{\sqrt{2}} + 0\right) = \frac{\pi}{12}\left(1+2\sqrt{2}\right) \approx 1.0023.$