Controlling Air Conditioners

Direct Control for Demand Response
Architecture

Generators → Payment → ISO

Price forecast → LSE

Energy budget, time horizon

Measured total power → Target total power

Aggregate loads
Research Scope

Objective: A theory of operation for the LSE

Challenges:

1. How to design the target consumption as a function of price?

2. How to control so as to preserve privacy of the loads’ states?

3. How to respect loads’ contractual obligations (e.g. comfort range width $\Delta$)?
First layer: planning optimal consumption

- (Energy budget, Time horizon) = (E, T)
- ISO
  - Payment = $\int_0^T \hat{\pi}(t) P_{\text{total}}^\text{ref}(t) \, dt$
- Forecasted ambient $\hat{\theta}_a(t)$
- Forecasted price $\hat{\pi}(t)$

Real-time ambient temperature $\theta_a(t)$

Second layer: setpoint control

- LSE
  - $P_{\text{total}}^\text{ref}(t)$
- $e(t)$
- Setpoint controller
  - $v(t)$
- $P_{\text{total}}(t)$
- Aggregate power sensor
- Smart thermostat
  - $\Delta v(t)$
- AC population
First Layer: Planning Optimal Consumption

\[ \text{minimize} \quad \int_0^T \frac{P}{\eta} \hat{\pi}(t) \left( u_1(t) + u_2(t) + \ldots + u_N(t) \right) \, dt \]

subject to

1. \[ \dot{\theta}_i = -\alpha_i \left( \theta_i(t) - \hat{\theta}_a(t) \right) - \beta_i P u_i(t) \quad \forall \ i = 1, \ldots, N, \]

2. \[ \int_0^T (u_1(t) + u_2(t) + \ldots + u_N(t)) \, dt = \tau = \frac{\eta E}{NP} (< T, \text{given}) \]

3. \[ L_{i0} \leq \theta_i(t) \leq U_{i0} \quad \forall \ i = 1, \ldots, N. \]

Optimal consumption: \[ P_{\text{ref}}^*(t) = \frac{P}{\eta} \sum_{i=1}^{N} u_i^*(t) \]
Second Layer: Real-time Setpoint Control

$$P^*_\text{ref}(t) = \frac{P}{\eta} \sum_{i=1}^{N} u^*_i(t), \quad e(t) = P^*_\text{ref}(t) - P_{\text{total}}(t),$$

$$v(t) = k_p e(t) + k_i \int_0^t e(\zeta) d\zeta + k_d \frac{d}{dt} e(t), \quad \frac{ds_i}{dt} = \Delta_i,$$

$$L_{it} = U_{i0} \land [L_{i0} \lor (s_i(t) - \Delta_i)], \quad U_{it} = L_{i0} \lor [U_{i0} \land (s_i(t) + \Delta_i)].$$
Boundary Control: Deadband → Liveband
Simulation: 500 homes + ERCOT DA price
How Can the LSE Price A Contract

![Graph showing the relationship between increase in per day cost ($) and Δ (°C).]
Details in


4. A. Halder, X. Geng, F.A.C.C. Fontes, P.R. Kumar, and L. Xie, "Deterministic and Stochastic Optimal Control of Thermal Inertial Loads", *working manuscript, available upon request*. 
Thank You
Backup Slides
First Layer: "discretize-then-optimize"
First Layer: "discretize-then-optimize"

Numerical challenges for MILP and LP

Solution: continuous time $\leadsto$ PMP w. state inequality constraints
Differential Privacy Preserving Sensing

\[
LSE \quad \tilde{P}_{\text{total}}(t) \\
\quad \nu \quad \tilde{P}_1(t) \quad \tilde{P}_i(t) \quad \tilde{P}_N(t) \\
\quad \quad \quad p \quad \quad \quad \quad \quad p \quad \quad \quad \quad \quad p \\
\quad n_1 \quad \quad \quad n_i \quad \quad \quad \quad \quad n_N \\
\quad \quad \quad P_1(t) \quad \quad \quad P_2(t) \quad \quad \quad P_N(t) \\
\quad \quad \quad \quad \quad AC 1 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad AC i \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad AC N
\]
Houston Data for August 2015

\(\hat{\theta}_a(\omega,t) \, (^\circ \text{C})\)

\(\theta_a(\omega,t) \, (^\circ \text{C})\)

\(\hat{\pi}_{DA}(\omega,t) \, ($/\text{MWh})\)

\(P_{\text{ref total}}(\omega,t) \, (\text{kW})\)
Limits of Control Performance