Control Objectives

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Recap: state space

Set in which the collection of state variables belong to

We saw examples: interval, infinite real plane, infinitely long cylinder, torus/bagel/doughnut

(infinite real plane: $\mathbb{R}^2$)

Example: state space for spring pendulum

State space:

Process state variables: $(\theta, \omega, \ell, v) =$ (angle, angular velocity, length, rate of change of length) in (rad, rad/s, ft, ft/s) at time t

Fixed points/equilibria: $\left(0,0,\ell_0 - \dfrac{mg}{k},0\right), \quad \left(\pi,0,\ell_0 - \dfrac{mg}{k},0\right)$

ℓ_0 is the rest length

Example: state space for spring pendulum

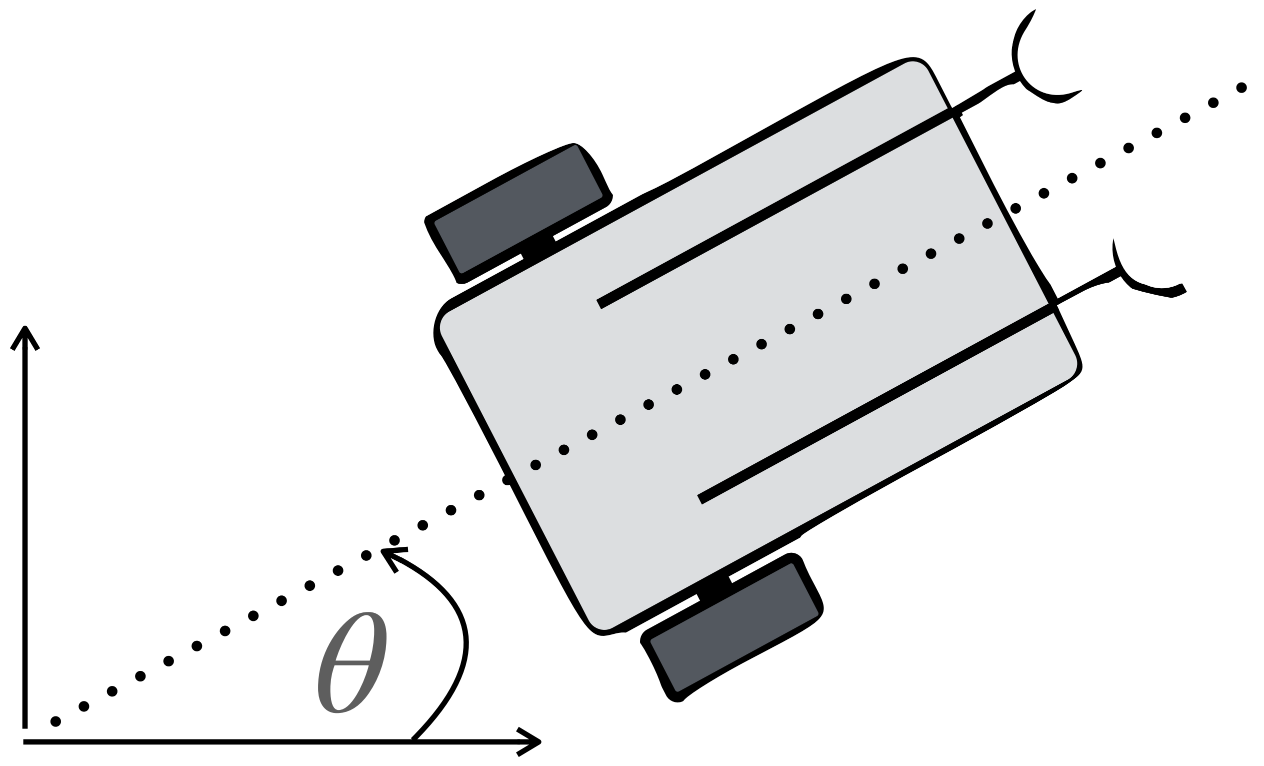
State space: $[-\pi, \pi] \times \mathbb{R}^3$

Process state variables: $(\theta, \omega, \ell, v)$ = (angle, angular velocity, length, rate of change of length) in (rad, rad/s, ft, ft/s) at time t

Fixed points/equilibria: $\left(0,0,\ell_0 - \frac{mg}{k},0\right)$, $\left(\pi,0,\ell_0 - \frac{mg}{k},0\right)$

ℓ_0 is the rest length

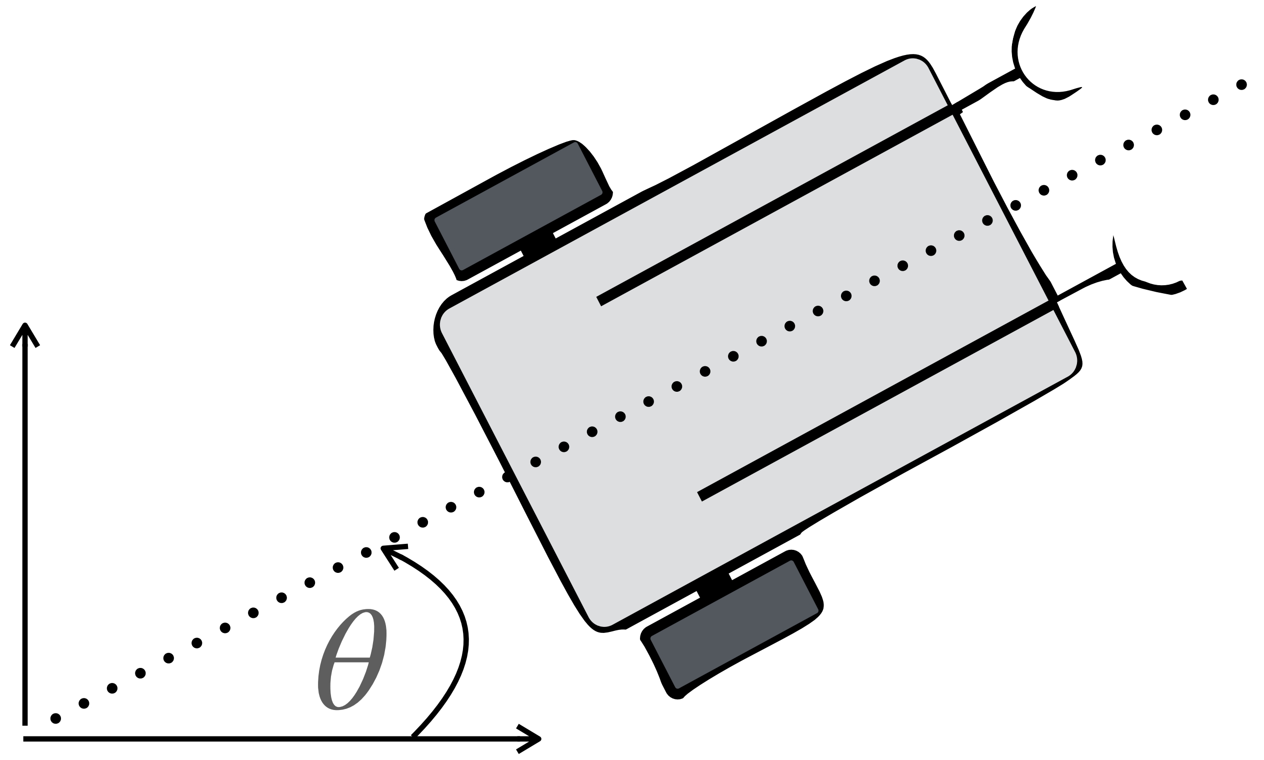
Example: state space for wheeled mobile robot



State space:

Process state variables: (x_1, x_2, θ) = (horizontal position, vertical position, heading angle) in (ft, ft, rad) at time t

Example: state space for wheeled mobile robot



State space: $\mathbb{R}^2 \times [0,2\pi)$

Process state variables: $\left(x_1, x_2, \theta\right)$ = (horizontal position, vertical position, heading angle) in (ft, ft, rad) at time t

Control objectives

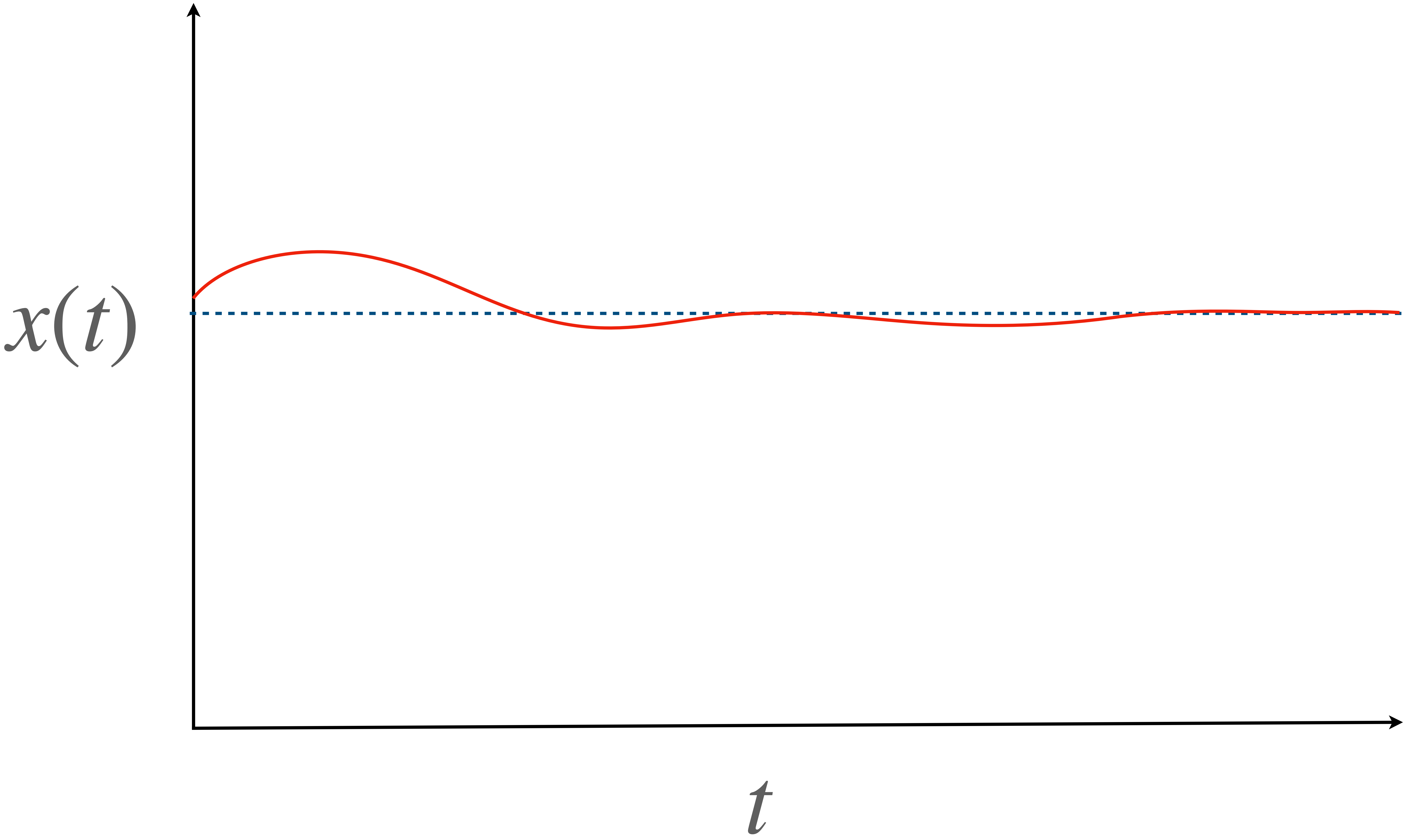
Controller is an **algorithm**

Designing this algorithm is called **control design** or **control synthesis**

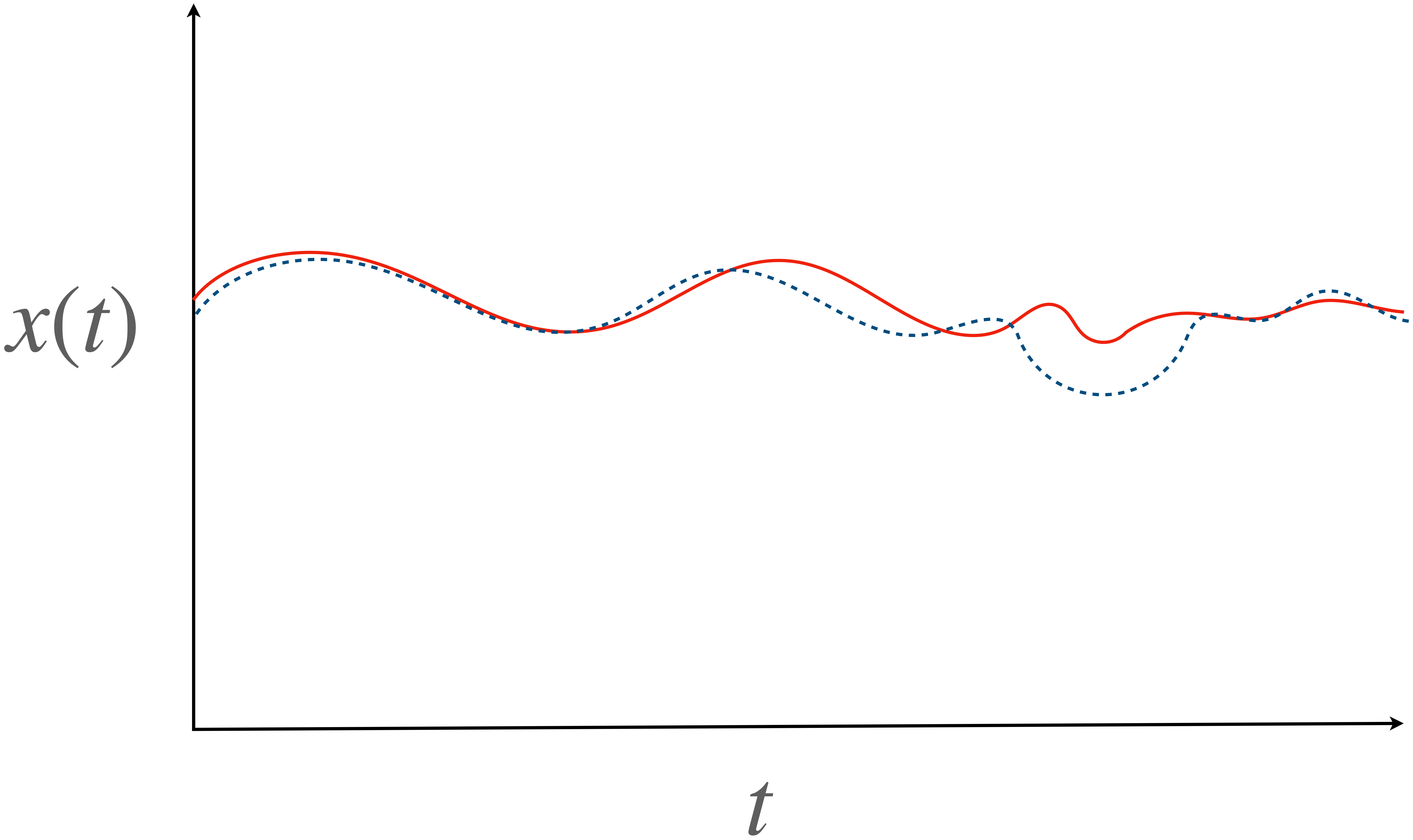
Different objectives lead to different types of algorithms

Examples of different objectives: regulation, tracking

Regulation: finite vs infinite horizon



Tracking: finite vs infinite horizon



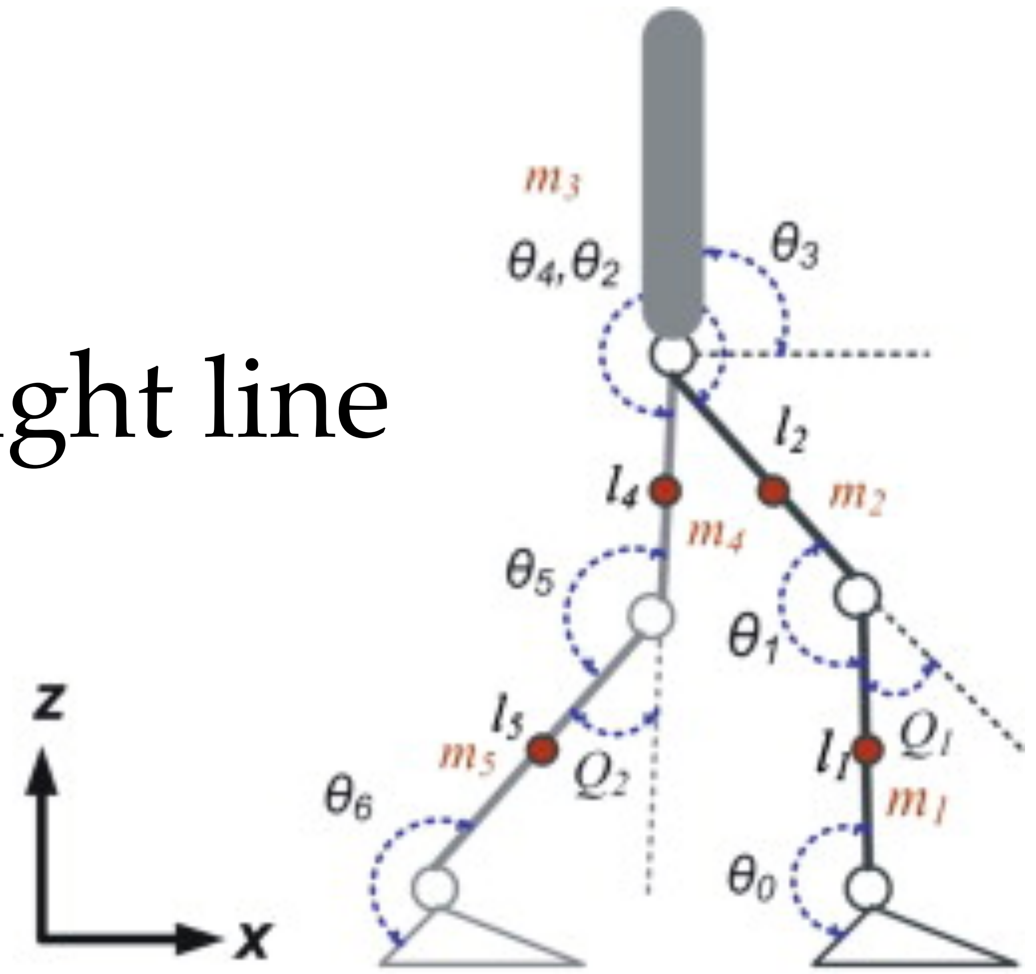
Fill in the blanks: regulate or track?

Artificial pancreas: ________________ blood glucose concentration

Car cruise control: ________________ car speed to the speed limit

Wheeled mobile robot: ________________ a given desired path avoiding obstacles

Bipedal walking robot: ________________ state variables to walk along a straight line



Different subdisciplines for different control design philosophies

Classical linear/nonlinear control: regulate or track signals, typically concerns with $t \to \infty$

Optimal control: optimization + control $\rightsquigarrow$ regulate or track while minimizing some loss/cost

Stochastic control: optimal control with noise and statistical uncertainties

Adaptive control: optimal control with unknown dynamics: learning + control

Often mix these: e.g., stochastic + adaptive, etc.

Frequent intersection with other disciplines

Dynamic games: game theory + optimal control

Machine learning: learning for control, control for learning

Control of networked systems: graph theory + control