Linear Feedback

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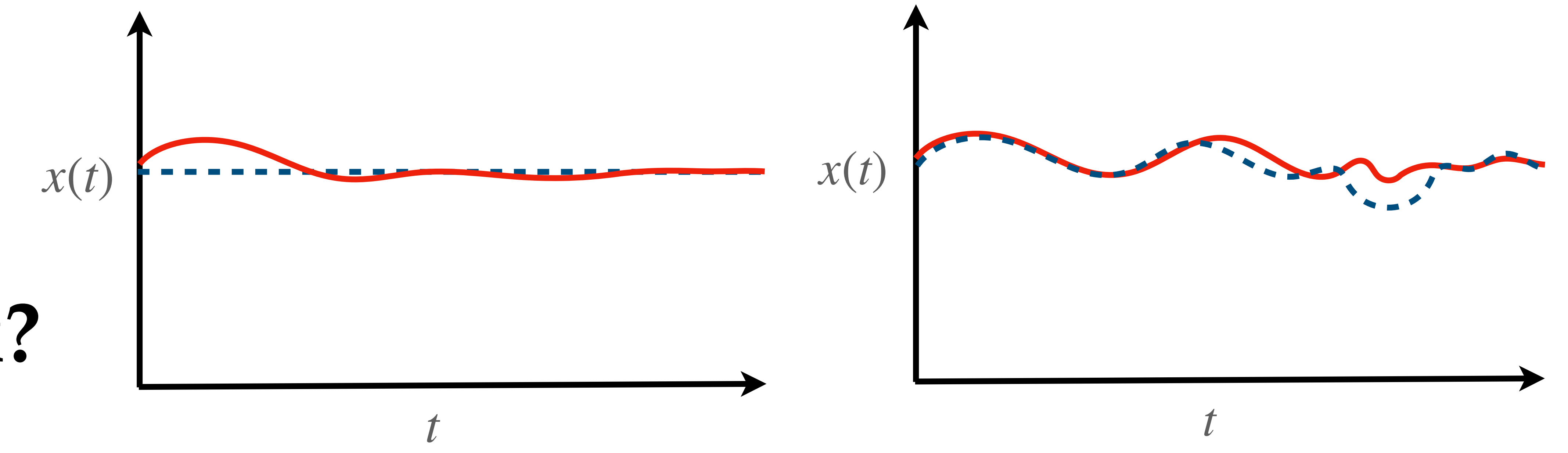
Recap: control objectives

Controller is an algorithm

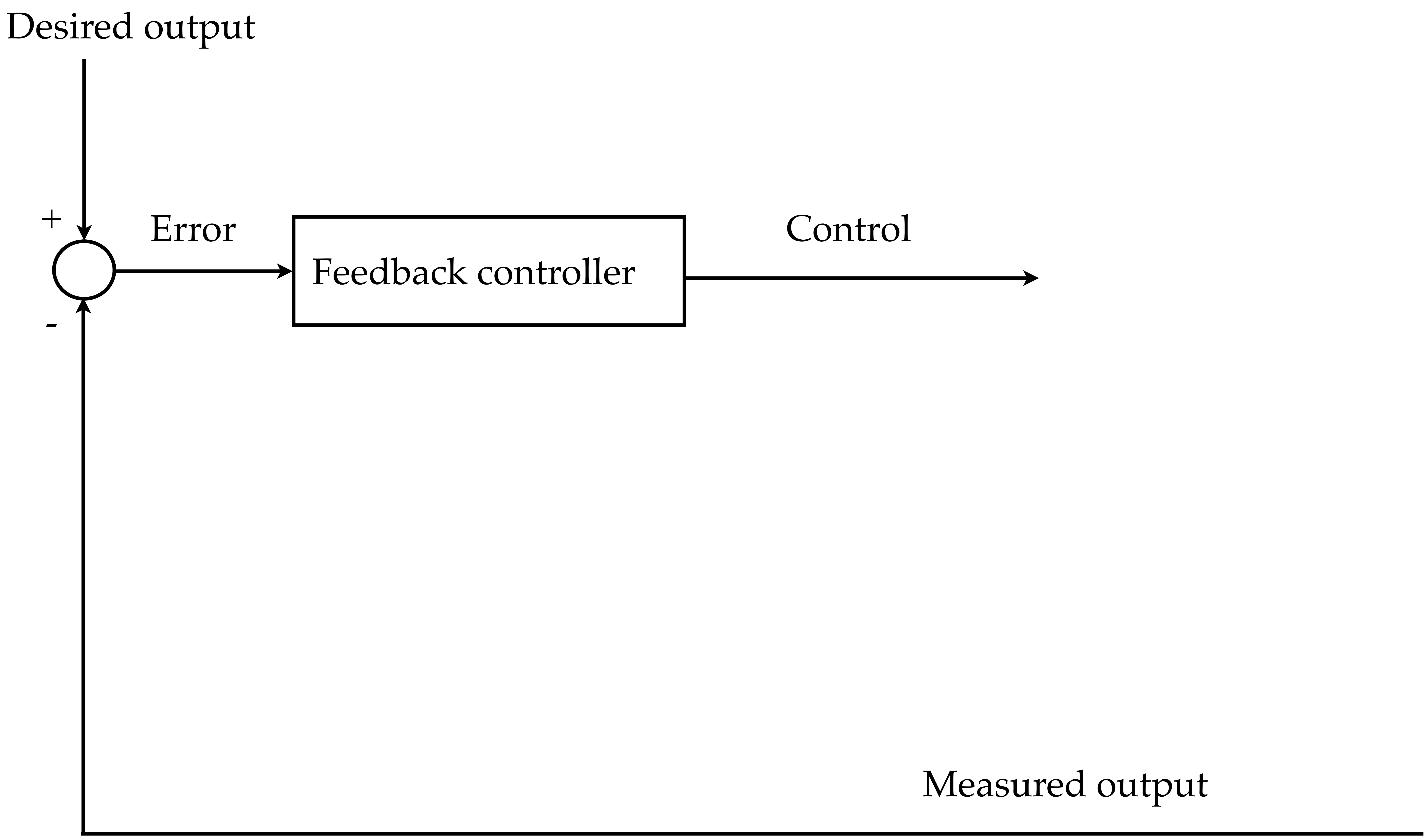
Designing control algorithm: **what** and **how**

What should the control algorithm do: **regulate? track?**

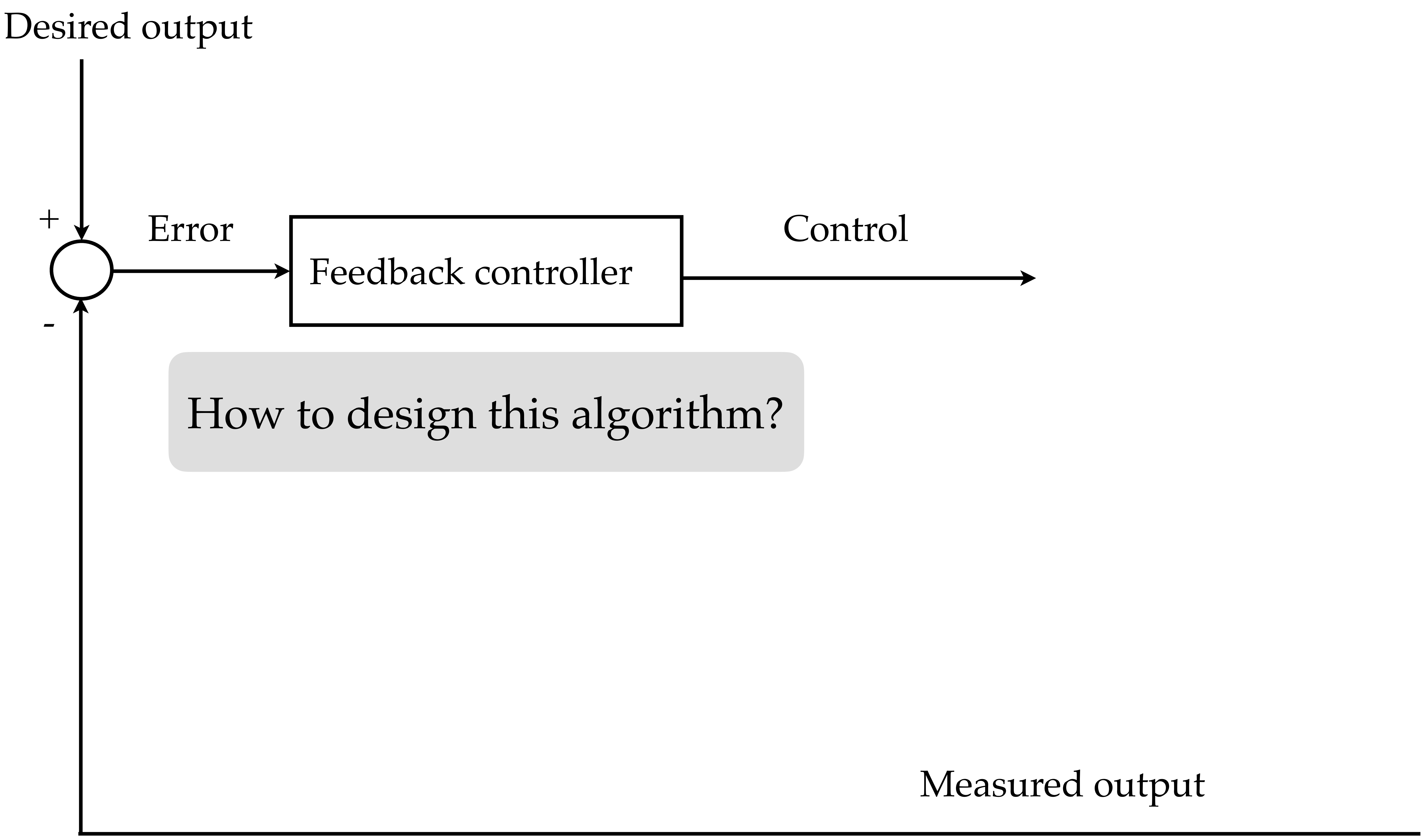


Today: **How** to design a regulator (controller that achieves regulation) or tracker (controller that achieves tracking)?

Recap: from our block diagrams



Recap: from our block diagram



How to design this algorithm?

A simple control algorithm: linear feedback

Suppose the feedback is only **one** output y AND we have only **one** control u

Control is **proportional** to error

Then control

$$u(t) = \underbrace{k}_{\text{constant}} \underbrace{\left(\underbrace{y_d(t)}_{\text{desired output}} - \underbrace{y(t)}_{\text{measured output}} \right)}_{\text{error at time } t}$$

This is same as the **proportional control your learnt in the robotics course**

Mathematically, this is same saying "control is a **linear** function" of the error

A simple control algorithm: linear feedback

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Control is **proportional** to error

Then control

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more (less) error $\rightsquigarrow$ more (less) corrective action

This is same as the **proportional control your learnt in the robotics course**

Mathematically, this is same saying "control is a **linear** function" of the error

A simple control algorithm: linear feedback

Suppose the feedback comprises of **multiple** (for example, two) output signals y_1, y_2
AND we have only **one** control u

Control is **linear** function of the errors

Then control

$$u(t) = \underbrace{k_1}_{\text{constant}} \underbrace{\left(\underbrace{y_{1d}(t)}_{\text{desired first output}} - \underbrace{y_1(t)}_{\text{measured first output}} \right)}_{\text{error in first output at time } t} + \underbrace{k_2}_{\text{constant}} \underbrace{\left(\underbrace{y_{2d}(t)}_{\text{desired second output}} - \underbrace{y_2(t)}_{\text{measured second output}} \right)}_{\text{error in second output at time } t}$$

Designing the controller reduces to finding the constants/gains k_1, k_2

A simple control algorithm: linear feedback

Suppose the feedback comprises of **multiple** (for example, two) output signals y_1, y_2
AND we have **multiple** (for example, two) controls u_1, u_2

Controls are **linear** functions of errors

Then controls

A simple control algorithm: linear feedback

Suppose the feedback comprises of **multiple** (for example, two) output signals y_1, y_2
AND we have **multiple** (for example, two) controls u_1, u_2

Then controls

Controls are **linear** functions of errors

$$u_1(t) = \underbrace{k_{11}}_{\text{constant}} \underbrace{\left(\underbrace{y_{1d}(t)}_{\text{desired first output}} - \underbrace{y_1(t)}_{\text{measured first output}} \right)}_{\text{error in first output at time } t} + \underbrace{k_{12}}_{\text{constant}} \underbrace{\left(\underbrace{y_{2d}(t)}_{\text{desired second output}} - \underbrace{y_2(t)}_{\text{measured second output}} \right)}_{\text{error in second output at time } t}$$

$$u_2(t) = \underbrace{k_{21}}_{\text{constant}} \underbrace{\left(\underbrace{y_{1d}(t)}_{\text{desired first output}} - \underbrace{y_1(t)}_{\text{measured first output}} \right)}_{\text{error in first output at time } t} + \underbrace{k_{22}}_{\text{constant}} \underbrace{\left(\underbrace{y_{2d}(t)}_{\text{desired second output}} - \underbrace{y_2(t)}_{\text{measured second output}} \right)}_{\text{error in second output at time } t}$$

Recap: wheeled mobile robot from Lecture 5

MATLAB exercise: design a linear feedback controller for tracking

Ignore noise: assume $w(t) = 0$, $v_1(t) = 0$, $v_2(t) = 0$

Choose some meaningful initial condition

Choose some meaningful desired path

Use for loop over the discrete time index

Experiment with different choices of control gains