Filtering

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Recap: probabilistic uncertainties and state estimation

Estimation problem: estimate process states given noisy measurements + imperfect models

Filter = algorithm that solves the estimation problem

Filter must work in real time, cannot do batch processing of data

Filter must guarantee "best" estimate, typically "best" = minimum mean squared error (MMSE)

Recap: probabilistic uncertainties and state estimation

Common way to measure "best": minimum mean squared error (MMSE)

Find $\hat{x}(t)$ that minimizes

Expected value of $\left[(x(t) - \hat{x}(t))^2 \mid \text{history of measurements up until time } t\right]$

It turns out that the minimizer is what is called "conditional expectation of the process state"

Many algorithms: Kalman filter, particle filters

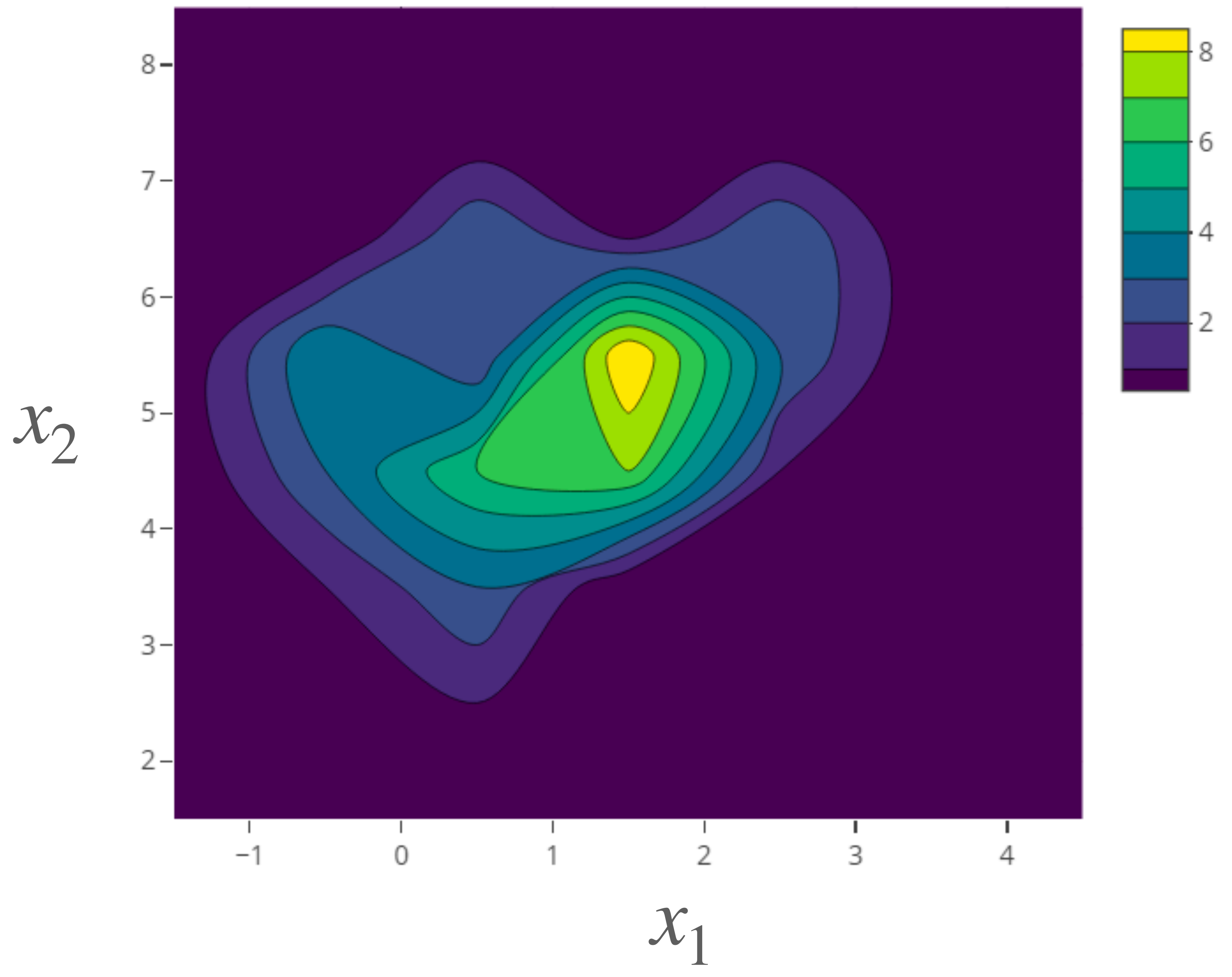
How do the filtering algorithms work: conceptually

Conditional expectation of the process state, that is, the expected state given the history of noisy measurements, **is random/probabilistic/stochastic**

Thus, the conditional expectation of the process state has a probability distribution

This distribution is changing with time *t*

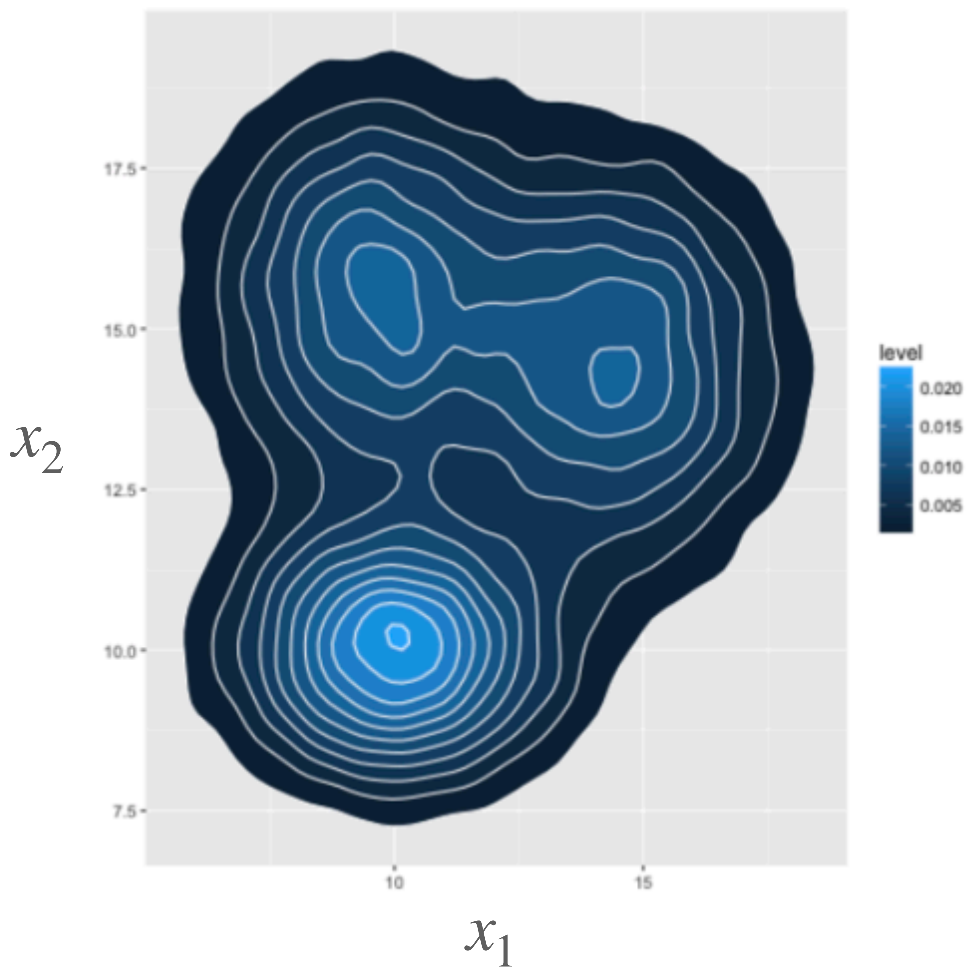
This distribution is called the "posterior" distribution



How do the filtering algorithms work: conceptually

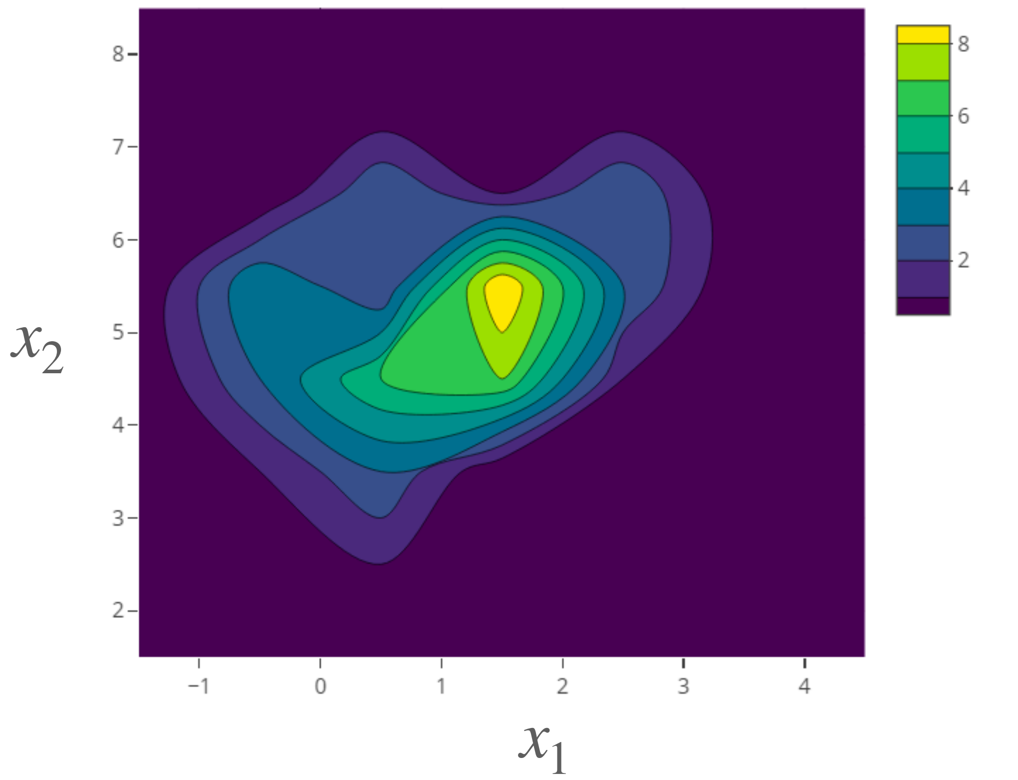
Two step computation: **prior** state distribution ⇝ **posterior** state distribution

Step 1: Prediction — compute the **prior** state distribution at time t



Use the imperfect process model

Step 2: Correction — update the **prior** from step 1 to **posterior** state distribution at time t



Use the imperfect measurement model + (noisy) measured data

Application to radar tracking

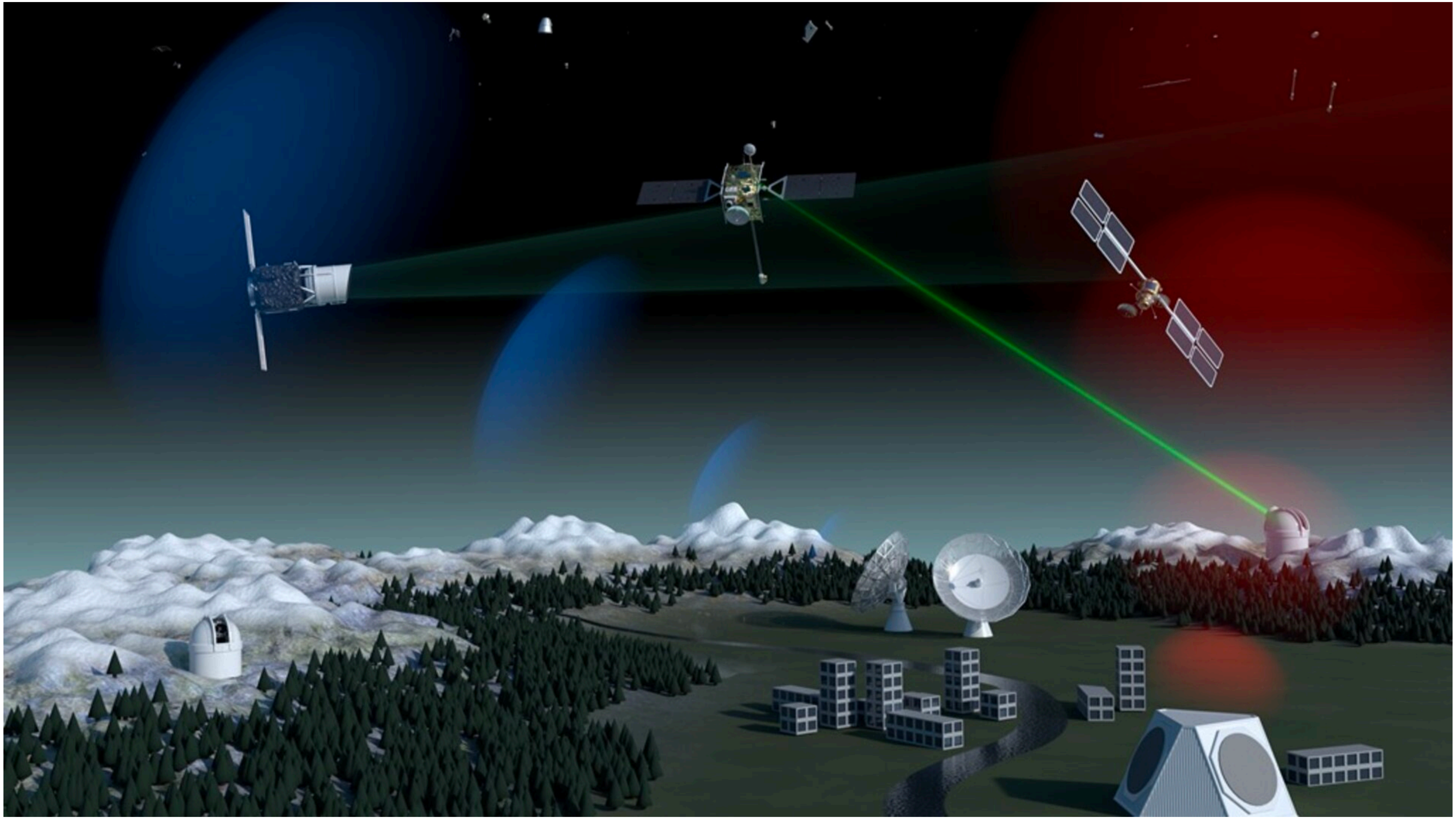


Image credit: European Space Agency

Application to tracking space debris

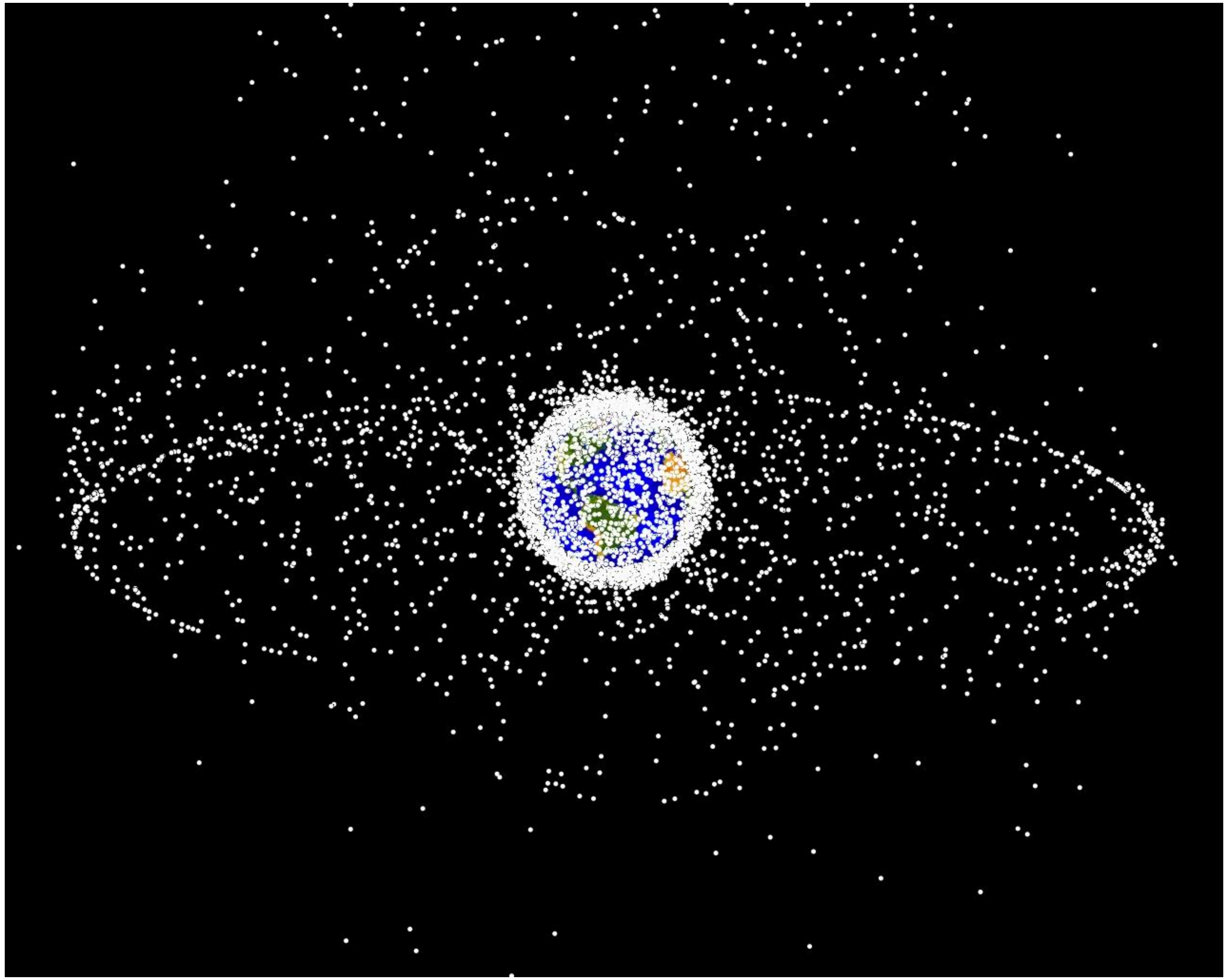
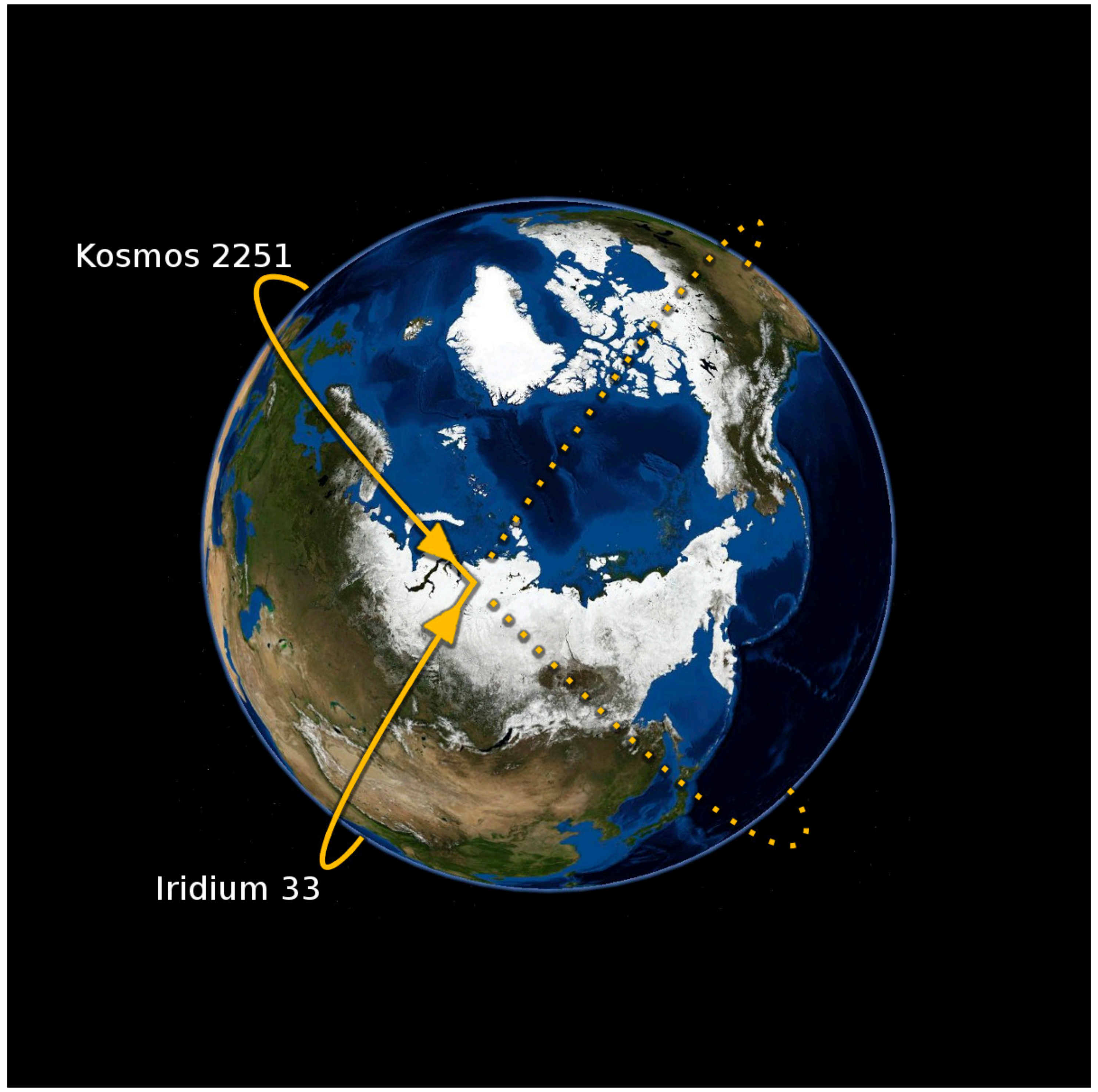


Image credit: NASA

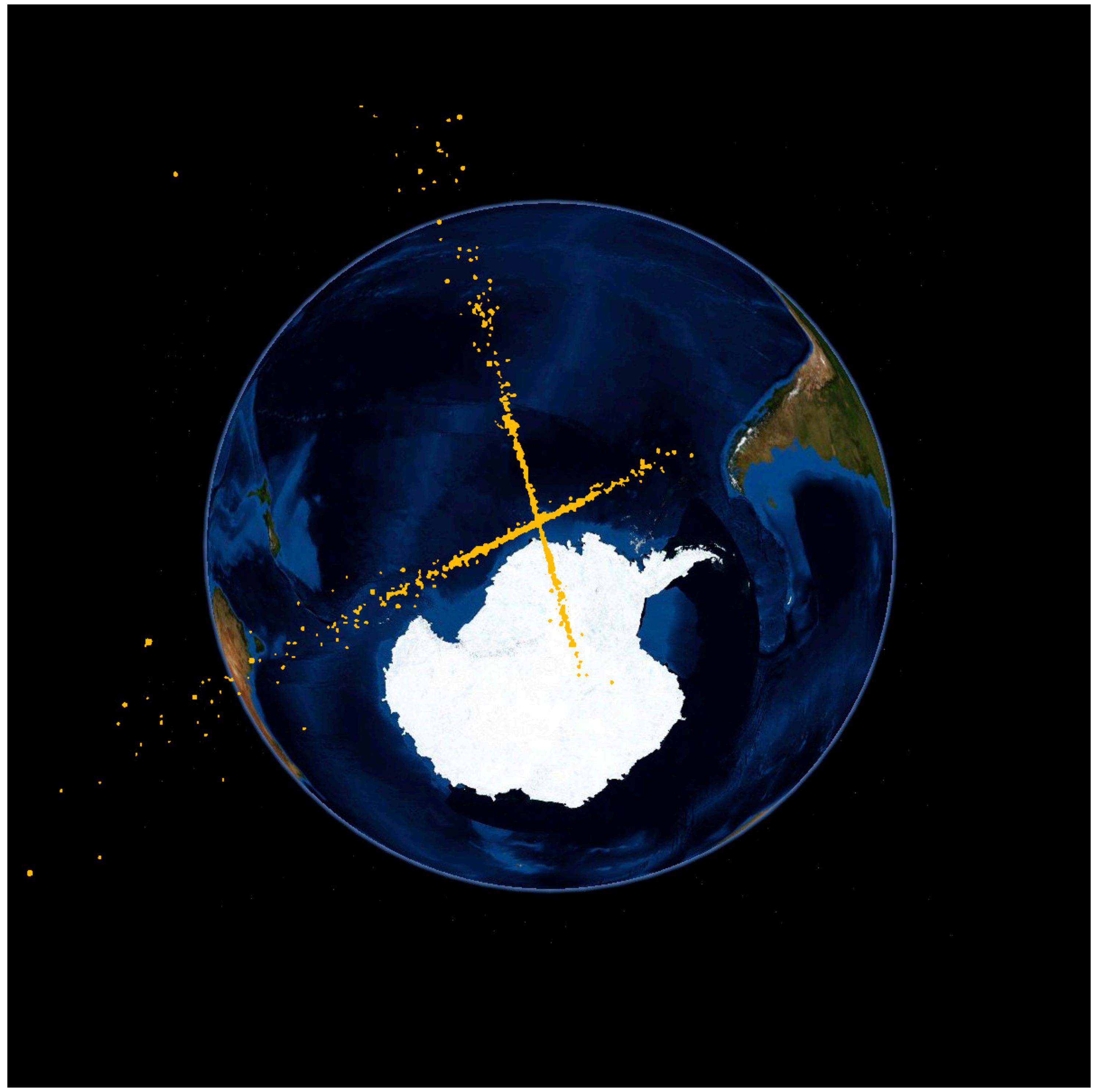
Animation by the European SpaceAgency:

https://www.esa.int/ESA_Multimedia/Videos/2019/02/Distribution_of_space_debris_in_orbit_around_Earth

2009 Collision between Iridium 33 and Kosmos 2251



20 min after collision



50 min after collision

Image credit: Wikipedia