Linear versus nonlinear

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Recap: Oscillation

Period 2 cycle or oscillation: definition and example

Period 2 cycle or oscillation: how to analyze

Example: Process dynamics

$$x_1(t+1) = \frac{1}{2}x_1(t) - \frac{\sqrt{3}}{2}x_2(t)$$

$$x_2(t+1) = \frac{\sqrt{3}}{2}x_1(t) + \frac{1}{2}x_2(t) + u$$

control

Question: What is/are the fixed point(s) for the **uncontrolled** dynamics (when $u = 0$)?

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Answer: Still unique fixed point $(x_1, x_2) = (0,0)$.

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Answer: Feedback control since it is feeding back a function of the state variables

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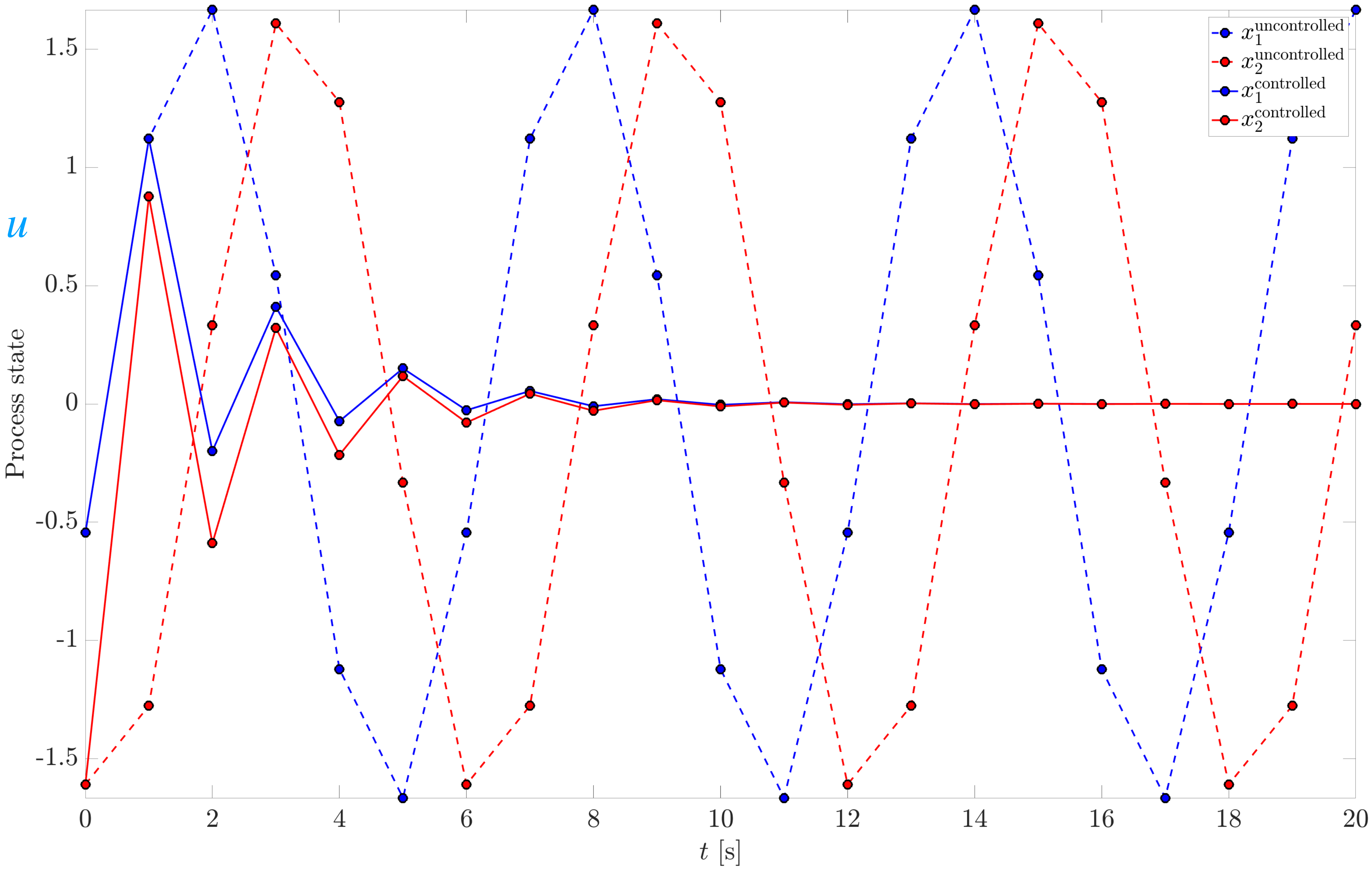

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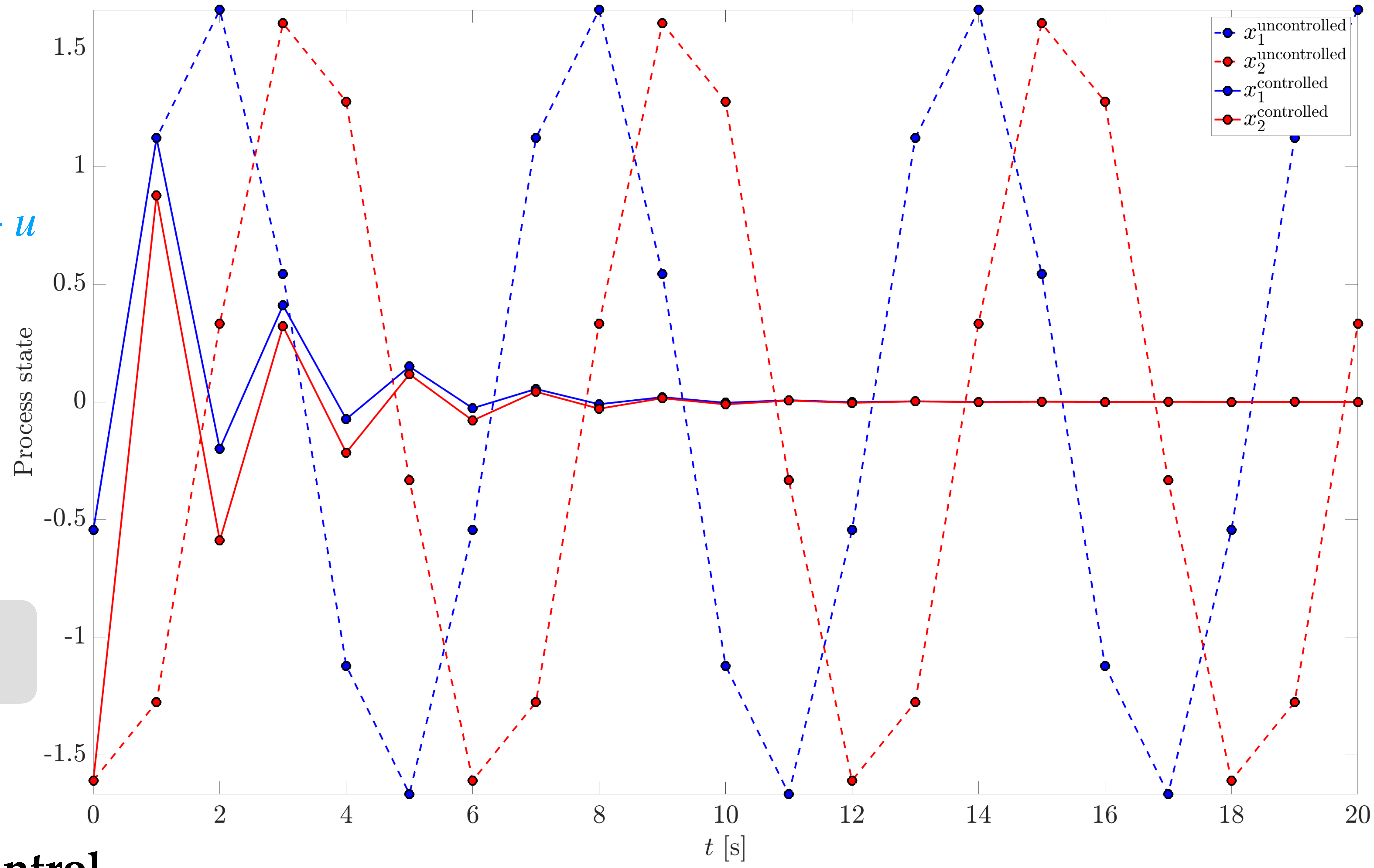
$u = 0 \rightsquigarrow$ Oscillation

$u = -x_1 - x_2 \rightsquigarrow$ GAS

Stabilizing state feedback control



Linear versus nonlinear dynamics

Recall: general discrete time process dynamics is of the form $x(t+1) = f(x(t))$

We say that the function f is linear if and only if $f(ax+by) = af(x) + bf(y)$ for any real a, b

Function f is **nonlinear** $\Leftrightarrow$ Function f is NOT linear

$\Updownarrow$

Dynamics $x(t+1) = f(x(t))$ is **nonlinear**

Generalizes for multiple variables x_1, x_2, x_3 etc.

Which of the following dynamics are linear, which one are nonlinear?

Example 1

$$x(t+1) = 3.2\, x(t)\big(1 - x(t)\big)$$

Example 2, set $u = 0$

$$x_1(t+1) = \frac{1}{2}x_1(t) - \frac{\sqrt{3}}{2}x_2(t)$$

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Example 2, now with $u = -x_1 - x_2$

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← **Nonlinear**

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← **Linear**

Example 2, now with $u = -x_1 - x_2$ ← **Linear**