Understanding the Geometry of Integrator Reach Sets for Robotics Applications

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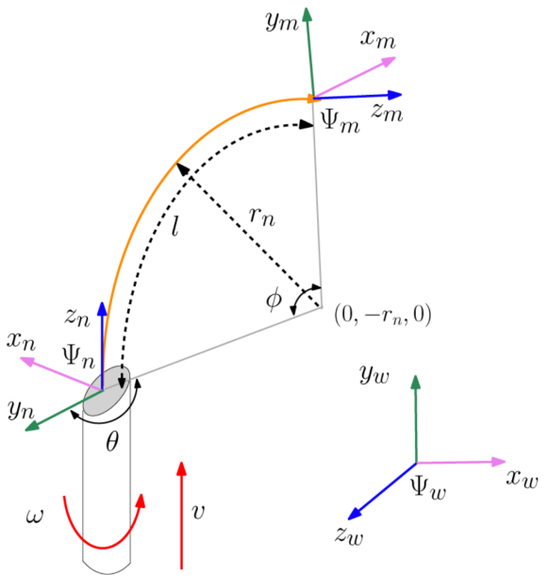
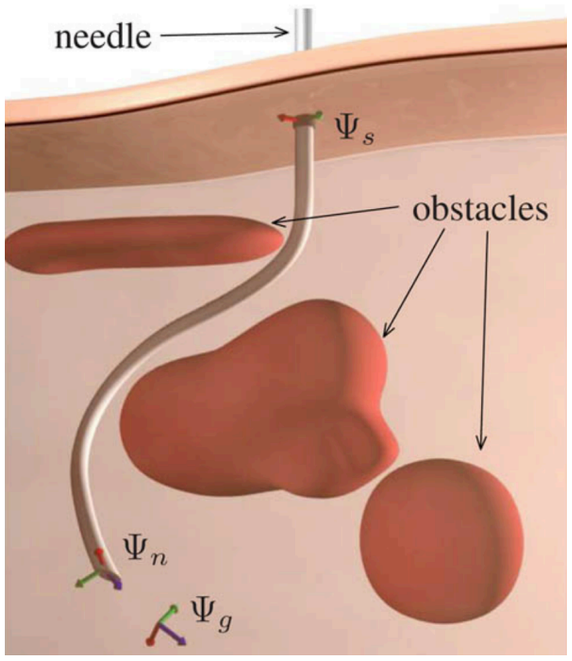


Credit: Patil and Alterovitz, *2010*



Credit: Duindam et al. *2009*

Reach set:

Where can the robot be at a future time subject to the dynamics and current knowledge of uncertainties

d dimensional reach set volume at time t for $|u| \leq \mu$:

$$\mathrm{vol}(\mathcal{R}(\{\boldsymbol{x}_0\}, t)) = (2\mu)^d t^{\frac{d(d+1)}{2}} \prod_{k=1}^{d-1} \frac{k!}{(2k+1)!}$$

Diameter of the reach set:

$$\mathrm{diam}(\mathcal{R}(\{\boldsymbol{x}_0\}, t)) = 2\mu \sqrt{\sum_{j=1}^{d} \left(\frac{t^j}{j!}\right)^2}$$