A Control System Framework for Privacy Preserving Demand Response of Thermal Inertial Loads

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Research Objective

A theory for operation of the load serving entity (LSE) to enable demand response by controlling the aggregate power consumption for a population of thermostatically controlled loads (TCLs) such as residential air conditioners.

Research Challenges

1. How to design the *reference* total power trajectory as a function of the forecasted price of energy?
2. The room temperature, setpoint, and ON/OFF binary state of any individual TCL cannot be measured for privacy reasons.
3. The LSE may have different contractual obligations for different TCLs in terms of their comfort ranges.

Key Question: What is the *optimal* plan for the LSE to schedule the purchase of power? Also, how to control the TCLs in real-time to track the reference total power, while respecting privacy and comfort range constraints?

Idea: Adjust setpoints to meet the optimized target consumption.

Proposed Architecture: A Two Layer Approach

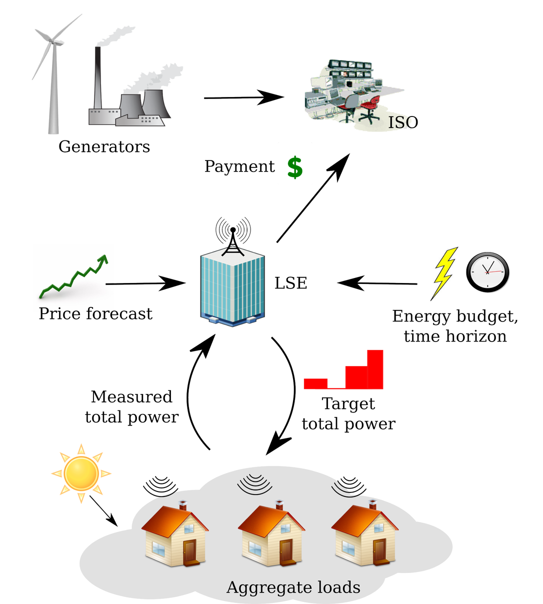

Fig. 1. Architecture of the proposed demand response system.

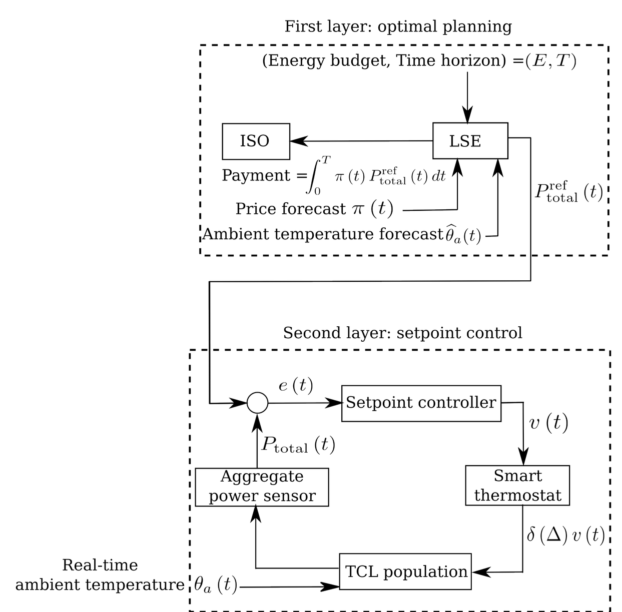

Fig. 2. Block diagram for the two layer control framework.

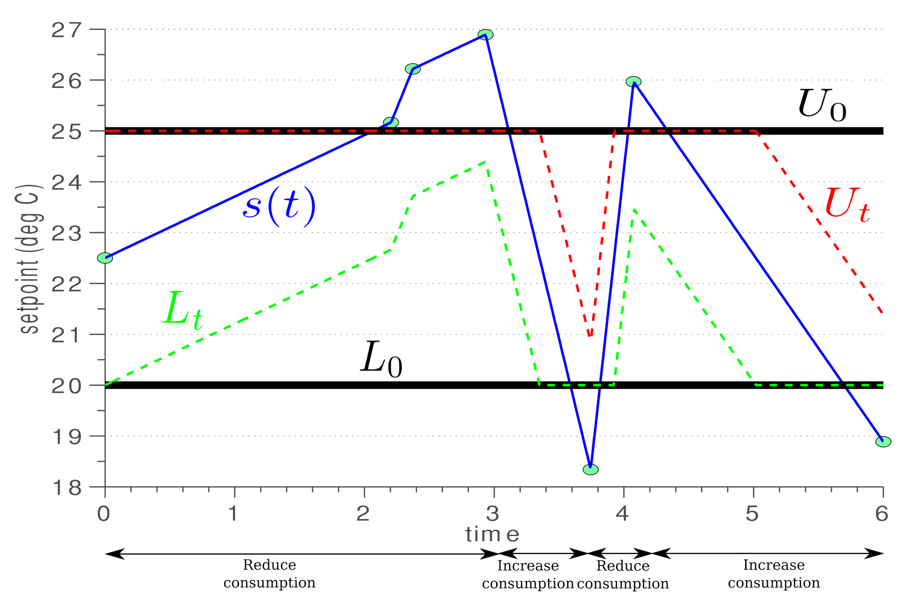

Fig. 3. Setpoint control with comfort range constraint to track optimal reference power trajectory.

Formulation

First layer: optimal planning of target consumption

$$\underset{\{u_1(t),\ldots,u_N(t)\}\in\{0,1\}^N}{\text{minimize}} \int_0^T P\pi(t)\left(u_1(t)+u_2(t)+\ldots+u_N(t)\right)\,\mathrm{d}t,$$

subject to

(1) $\dot{\theta}_i = -\alpha\left(\theta_i(t) - \widehat{\theta}_a(t)\right) - \beta P u_i(t) \qquad \forall\, i = 1,\ldots,N,$

(2) $\displaystyle\int_0^T \left(u_1(t)+u_2(t)+\ldots+u_N(t)\right)\,\mathrm{d}t = \tau \doteq \frac{E}{P} (< T, \text{given})$

(3) $L_0^{(i)} \leq \theta_i(t) \leq U_0^{(i)} \qquad \forall\, i = 1,\ldots,N.$

Second layer: setpoint control

$$P^{*}_{\text{ref}}(t) = P\sum_{i=1}^{N} u_i^{*}(t), \qquad e(t) = P^{*}_{\text{ref}}(t) - P(t),$$

$$v(t) = k_p e(t) + k_i\int_0^t e(\tau)\mathrm{d}\tau + \frac{\mathrm{d}}{\mathrm{d}t}e(t), \qquad \frac{\mathrm{d}s_i}{\mathrm{d}t} = \Delta_i v(t),$$

$$L_t^{(i)} = L_0^{(i)} \vee \left(s_i(t) - \Delta_i\right), \qquad U_t^{(i)} = U_0^{(i)} \wedge \left(s_i(t) + \Delta_i\right).$$

Numerical Results

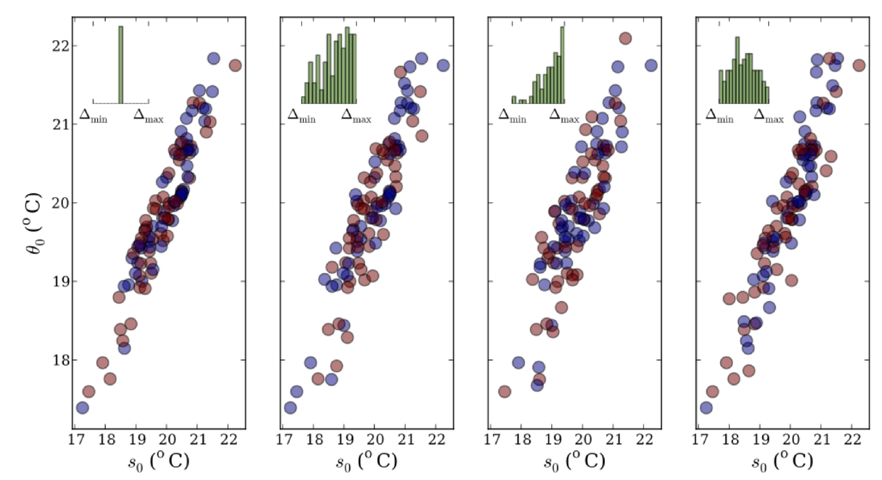

Fig. 4. 100 initial conditions $(s_0,\theta_0) \sim \mathcal{N}\left([20,20]^\top, \begin{bmatrix}1 & 0.5\\ 0.5 & 3\end{bmatrix}\right)$ where *red* denotes ON, and *blue* denotes OFF TCL. Four columns correspond to four different contract distributions (*inset histogram*). $\Delta_{\min} = 0.1^{\circ}$C, $\Delta_{\max} = 1.1^{\circ}$C.

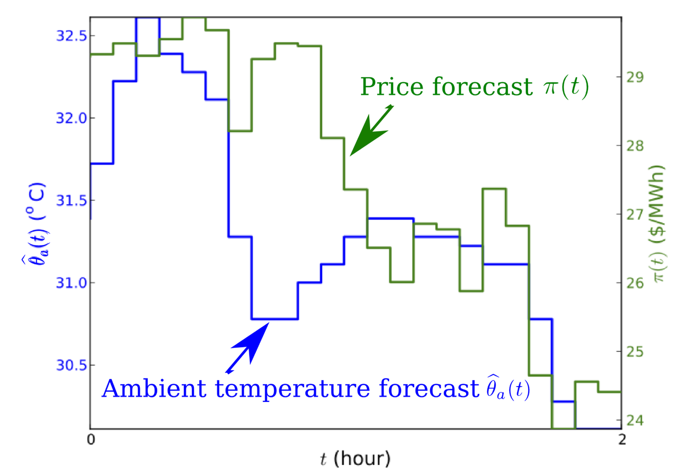

Fig. 5. Price and ambient temperature forecast for Houston on May 20, 2015, 4--6 pm.

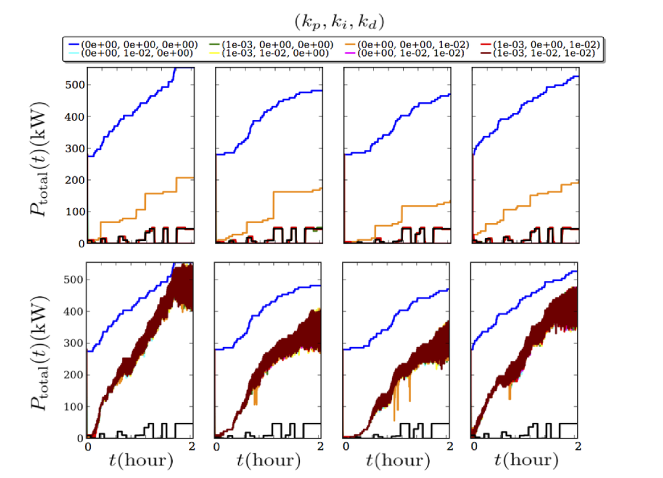

Fig. 6. PID controller tracks the total power consumption to the optimal reference consumption, with (*bottom row*) and without (*top row*) comfort constraints. Four columns are for four contract distributions, as in Fig. 4.

Theoretical Analysis for the Planning Problem

Why: Direct "discretize-then-optimize" approach leads to large scale mixed integer linear program (MILP), numerically difficult to solve.

How: Use Pontryagin's Maximum Principle (PMP) for the continuous time optimal control problem.

Optimality results from analysis:

- Constraints (1) and (2) active: $u^{*}_{12}(t) = \mathbf{1}_{\{\pi(t)\leq\pi^{*}\}}$, where $\pi^{*} \triangleq \inf\left(\widetilde{\pi} : \int_0^T \mathbf{1}_{\{\pi(t)\leq\widetilde{\pi}\}}\,\mathrm{d}t = \frac{\tau}{N}\right) \Rightarrow$ optimal *actions* are synchronized.
- Constraints (1), (2), and (3) active: $\theta^{*(i)}_{123}(t) = \Psi_{L_0^{(i)},U_0^{(i)}}\left(\theta^{*(i)}_{12}(t)\right)$, where $\Psi_{L,U}(\cdot)$ is the *two-sided or double Skorokhod map* in $[L,U]$.

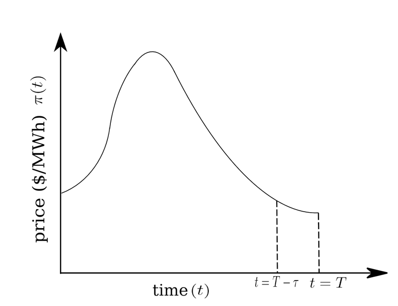

Fig. 7. Price forecast

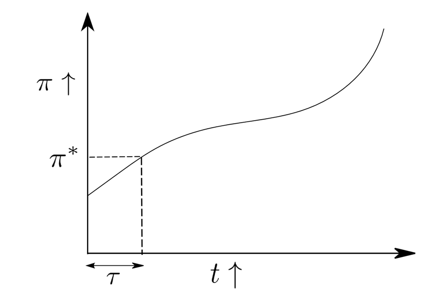

Fig. 8. Increasing rearrangement of price forecast

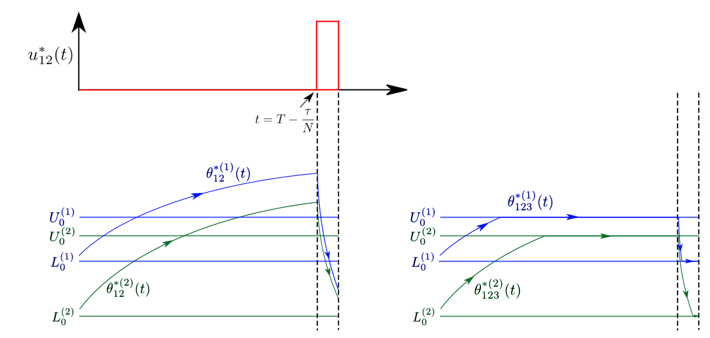

Fig. 9. Optimal control (*top*) and optimal temperature (state) trajectories (*bottom*)

Conclusions

- A simple framework for optimal demand response.
- Designs optimal target consumption using forecast.
- Tracks the designed target consumption in real-time.
- LSE does not need to know individual states ⇒ preserves privacy.

Acknowledgement

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References

[1] A. Halder, X. Geng, G. Sharma, L. Xie, and P.R. Kumar, "A Control System Framework for Privacy Preserving Demand Response of Thermal Inertial Loads". 6th *IEEE International Conference on Smart Grid Communications*, Miami, Florida, Nov. 2–5, 2015.