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AN ADAPTIVE FUZZY STATE NOISE DRIVEN EXTENDED KALMAN FILTER FOR REAL TIME ORBIT DETERMINATION

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Abstract

The orbit determination of the satellite, just after injection into the orbit, is crucial for the satellite tracking and planning of various immediate maneuvers required. Generally, Extended Kalman Filter (EKF), which is a suboptimal nonlinear implementation of linear Kalman filter, is employed for the real time orbit determination. However, the divergence of the EKF can not be ruled out, or at least a poor convergence may creep in even after employing various methods to make it adaptive by injecting noise. The divergence may occur due to errors in modeling the system, finite precision arithmetic and associated truncation/round-off errors and large errors can be attributed to a priori estimate and covariance. The artificial noise injection method, generally used for making the state covariance matrix positive definite, may not lead to proper convergence due to the problems mentioned above. In this paper a fuzzy state noise driven adaptive EKF which is based on spring- mass-damper analogy, has been proposed for orbit determination. The formulation makes the filter faster in convergence in the real time orbit determination application. A comprehensive simulation on PSLV-C1 data has been carried out to show the better convergence with the proposed fuzzy model.

INTRODUCTION

The problem of orbit determination [1] is of considerable significance for the early initiation of action for on-board operations including satellite tracking and control. The ability of Kalman filter [2] to handle noisy measurements and process noise in dynamics has made it a natural choice for orbit determination problem. Unlike simpler but time consuming batch processing techniques, the recursive nature of Kalman filter requires only the latest estimates, observations and corresponding variances of the errors to be retained, leading to significant saving in data storage. This makes it suitable for real time orbit determination. Extended Kalman filter (EKF), a suboptimal nonlinear implementation of linear Kalman filter, uses state or observation residuals to ensure the positive definiteness of the state covariance matrix. However, modeling errors and input statistics may degrade the numerical performance of EKF leading to the loss of positive definiteness of state covariance [3, 4]. Measurement bias and poor *a priori* estimate and covariance exacerbate the situation. Artificial noise injection technique is often employed to prevent the covariance matrix from diminishing too rapidly. Since the filter is suboptimal, the convergence is quite poor even when the noise covariance matrix is constructed based on random noise. It is required that Kalman gain must correspond to the error in the state. Hence, several adaptive noise models have been proposed [5, 6] to formulate the driving noise covariance matrix Q, either based on observation residuals or based on state residuals so as to keep the Kalman gain just suitable.

In this paper, the process noise covariance matrix has been formulated using fuzzy state residuals. The state residuals and their derivatives were computed by a moving window fuzzy regression method. A damping analogy was then used to construct the diagonal elements of the state noise covariance matrix to ensure the positive definiteness. As shown in the simulation results, a better convergence is achieved by the method proposed.

The rest of the paper is organized as follows. The next sections describe the orbit determination problem followed by a brief note on EKF. The adaptive fuzzy state noise driven filter is illustrated in the subsequent section. Finally simulation results are presented, followed by conclusion.

ORBIT DETERMINATION PROBLEM

Orbit determination is the process of estimating parameters which completely describe the motion of an orbiting satellite, artificial or natural, utilizing a set of observations gathered either from on-board sensors or from one or more ground stations [7, 8]. Precise determination of an orbit is crucial for planning orbit maneuvers, anticipating eclipses, search and rescue satellite aided tracking (SARSAT) [9], accurate orbit control [10] and efficient functioning of navigation satellite time and ranging (NAVSTAR) systems [11].

Unlike generalized orbit determination (GOD), preliminary orbit determination (POD) has several constraints [8] which necessitate a suitable filtering technique. Inaccuracy in the initial estimates of position and velocity, and the requirement to process the data in real time makes the filter selection even more crucial. The observational data consists of the range, range rate, azimuth and elevation over a number of time steps as recorded from one or more ground stations, starting from a specified Julian date.

EXTENDED KALMAN FILTER

The purpose of a filter is to fit an orbit to the observational data in order to achieve some specified criterion like maximum likelihood, minimum variance or least squares [12]. Since 1809, a large number of estimation techniques have been applied till date to the orbit determination problem. Table I, based on the listing in [7], summarizes the various estimators for the orbit determination problem.

Although linear Kalman filter (LKF) was applied successfully for satellite

Table I. Orbit determination filters and their applications

Filter	Type	Applications
Gaussian least squares differential correction (GLSDC)	Batch Iterative	Vision based navigation (VISNAV) sensor systems for spacecraft docking and formation flying [13], various non linear estimation problems [14]
EKF	Recursive	SARSAT [9, 15], LANDSAT-4 [16, 17], LEO satellite [18], HEO satellite [19], autonomous orbit determination using horizon scanner measurements [20]
Adaptive EKF	Recursive	Relative navigation system for low Earth orbit (LEO) formation estimation using carrier-phase differential GPS [21], re-entry problems [22]
Square root information Filter (SRIF)	Sequential	Apollo Lunar missions, Mariner 9 Mars orbiter, Mariner 10 Venus-Mercury space probe [23]
UD filter (UDF)	Recursive	Viking Mars, Voyager Jupiter spacecraft

orbit determination [27], it may not be adequate for problems involving very high degree of non-linearity. The EKF performs better than the LKF when large differences occur in the observational data.

Divergence in Kalman filters

Mathematical description of any dynamical system suffers from intrinsic modeling errors due to inadequate system knowledge and errors introduced in the process of linearization. The situation gets further aggravated by the machine dependent truncation / round-off errors. As a result the estimates drift away from the true states leading to a growth of residuals. This phenomenon, commonly termed as 'divergence', is characterized by statistical inconsistency between the actual estimation errors and the error covariance matrix resulted from the filter [8].

The causes of divergence in Kalman filters are summarized below:

1. Discrepancies between the mathematical model used to derive the filter equations and the actual condition under which filter must operate. These include neglecting higher order terms in gravitational potential and inaccurate estimation of the gravity coefficients.
2. Existence of the dynamic bias [24] due to errors in the ballistic coefficients.
3. Round-off errors caused by digital computer implementation. An illustrative example of Kalman filter divergence due to single precision arithmetic (IBM 7090) can be found in [12].
4. Large errors in the initial assumption of the state vector and its covariance.

5. Shift in the point of divergence, if any, due to the improper choice of state vector elements [25].

Examples of Kalman filter divergence are reported in [26, 27]. In the Mariner project, a mid-course correction sent by the ground controllers, was completely ignored by the on-board EKF as the filter believed that all states were well known and no correction was needed. Such a behavior of a Kalman filter is commonly termed as 'smugness' [27] and is exhibited when divergence occurs. As a result of this, the state covariance matrix *P*, and hence the Kalman gain matrix *K*, diminishes rapidly. Consequently, the observations do not improve the state estimates any further.

Divergence Control

To control the divergence of a Kalman filter, one must ensure the positive definiteness of the state covariance matrix *P* [28]. Various attempts have been made to meet this requirement. Chodas [15] attempted to diagonalize *P* at the end of each time step. Later Brown and McPherson [29] showed that, it is sufficient to diagonalize *P* at the end of the first estimation step only. Some attempts have also been made to keep the values of the off-diagonal elements of *P* below a threshold noise level. The threshold was either determined by the magnitude of the diagonal entries [8] or determined experimentally [30]. To combat the divergence due to dynamic biases, Schmidt [31] and Pines [32] have suggested modification of the filter equations without increasing the number of state variables. The often employed artificial noise injection [33, 34, 35] technique does not guarantee the positive definiteness of the state noise covariance matrix since large number of simulations is needed for adjusting the process noise.

Adaptation of EKF with driving noise formulation can ensure better convergence. Fig. 1 depicts various noise driven adaptive filtering techniques [36], employed to circumvent the divergence problem. The structural

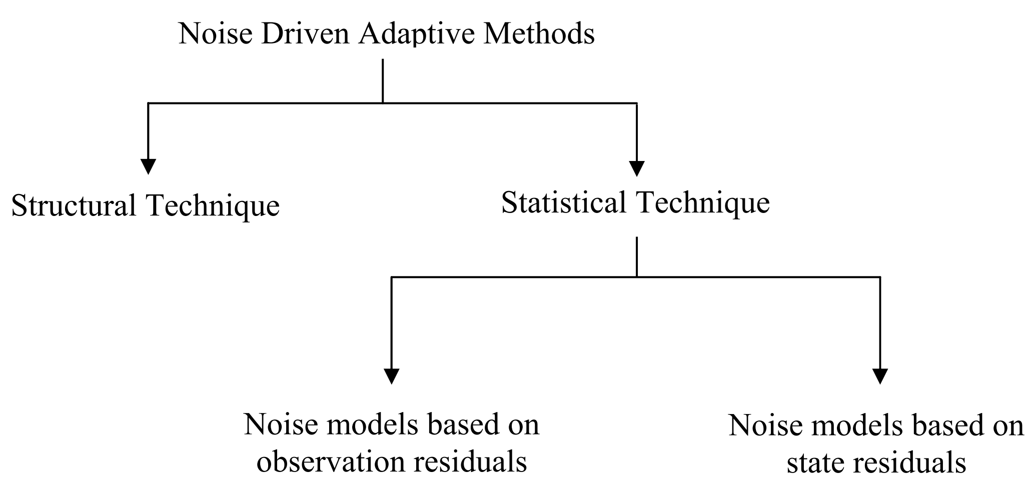

Fig. 1. Noise driven adaptive methods

technique estimates the un-modeled accelerations and is computation intensive. Statistical technique, in contrast, is suitable for real time orbit determination. Either observation residuals or state residuals can be used to formulate the process noise covariance matrix (*Q*). Wright [37, 38, 39] pointed out that, the process noise in orbit determination problem arises from the uncertainty in the force model and hence the process noise covariance matrix *Q* must be formulated as a function of system's physical processes. Based on the work of Kaula [40], he derived a process noise function for the errors in the gravity model. It has been shown [41] that, such physically defined filters are stable, robust and not fragile.

Both the statistical techniques attempt to derive a mathematical model of *Q* assuming it to be diagonal. In the first method, diagonal elements are constructed using the means and standard deviations of the observation residuals. Three such models are proposed in [6]. The second method uses state residuals and their derivatives, instead of observation residuals, to formulate *Q*. This approach eliminates the need to tune the filter (determination of noise participation constants), which was imperative in the former. Another drawback of the observation residual approach is that, the dimensional consistency of *Q* permits only the use of range and range rate residuals, neglecting the azimuth and elevation residuals, which are equally important [42]. A damping analogy has been used in [6] to formulate the state residual based noise model. The present paper proposes a fuzzified formulation for the same to ensure better convergence.

ADAPTIVE FUZZY STATE NOISE DRIVEN EXTENDED KALMAN FILTER

Based on the state residual noise model, process noise covariance matrix *Q* has been proposed [6] to be formulated as

$$Q = |\triangle\vec{x}\triangle\dot{\vec{x}}|\,\delta_{ij} \qquad (1)$$

where $\triangle\vec{x}$ is the state residual, $\triangle\dot{\vec{x}}$ is the derivative of the same and δ_{ij} stands for Kronecker delta. This model needs the computation of $\triangle\dot{\vec{x}}$ from the discrete state residuals available from the predictor step of the EKF. One simple way to compute the derivatives of the state residuals is to construct a fixed size data window and to fit a straight line segment through the state residuals inside that window using least square criteria. This operation has to be performed for each element of the state residual vector and the slopes can be found accordingly. Earlier results [42] have shown that, this approach of constructing driving noise covariance matrix is more robust than the usual artificial noise injection method where *Q* is constructed through ad hoc injection of random noise. It was observed that a better way of finding the derivatives of state residuals may improve the performance of the filter. Furthermore, there remained some ambiguity in finding the goodness of fit and in the fitting criterion. This prompted the authors to apply a linear fuzzy regression model to calculate the slope of the straight line fitted through the predicted state residuals inside the data window. The fitting criterion was to minimize the spread (fuzziness) of the state residuals instead of minimizing the least square errors. This allowed the EKF to capture the inherent ambiguity in the state residuals, which otherwise was not accounted.

Fuzzy Linear Regression Model

Since Tanaka et al [43] first proposed the fuzzy linear regression (FLR) problem in 1982, many attempts have been made to modify the basic algorithm to make it more versatile. FLR still remains a vibrant research topic receiving contributions from many research groups and individuals. FLR models can be broadly classified into two classes: possibilistic models and least square models. The two models differ in their optimization criteria.

Various possibilistic models can be found in [44, 45, 46]. Later Tanaka simplified the possibilistic FLR problem as interval linear regression problem [47, 48]. To achieve better performance, these models were coupled with neural network [49, 50, 51, 52], genetic algorithm [53, 54, 55] and Monte Carlo method [56]. Ordinary least square models and its variants for FLR can be found in [57-67]. For a more detailed discussion on the theoretical aspects of FLR, the interested reader can refer [68-90]. Various applications of FLR are reported in [91-101].

In this paper, Tanaka's original model of possibilistic FLR has been applied to find the slope of the state residuals inside a fixed data window. In conventional linear regression algorithm, the prime objective is to find the constants $a_0, a_1, a_2, ..., a_n$, where the dependent variable (state residual $\triangle\vec{x}$ in the present case) is expressed as a function of the independent variable (time t) as follows:

$$\triangle\vec{x} = f(t,a) = a_0 + a_1 t_1 + a_2 t_2 + ... + a_n t_n \quad (2)$$

In FLR, the dependent variable ($\triangle\vec{x}$) is allowed to be fuzzy (to account the imprecision in the orbit determination process and hence in the prediction of the EKF). Time, being the independent variable, is crisp. The regression constants are assumed to be fuzzy and need to be determined. Thus the regression equation for FLR becomes:

$$\triangle\tilde{\vec{x}} = f(t,\tilde{A}) = \tilde{A}_0 + \tilde{A}_1 t_1 + \tilde{A}_2 t_2 + ... + \tilde{A}_n t_n \quad (3)$$

where $\tilde{A}_i$ is the i^{th} fuzzy coefficient and $i = 0,1,2,...,n$. Here it is assumed that each of these fuzzy coefficients can be represented as a symmetric triangle. The membership function for $\tilde{A}_i$ is shown in Fig. 2, where c_i is the spread (half-width of the base) and p_i is the mid point of the base for fuzzy number $\tilde{A}_i$. Thus the

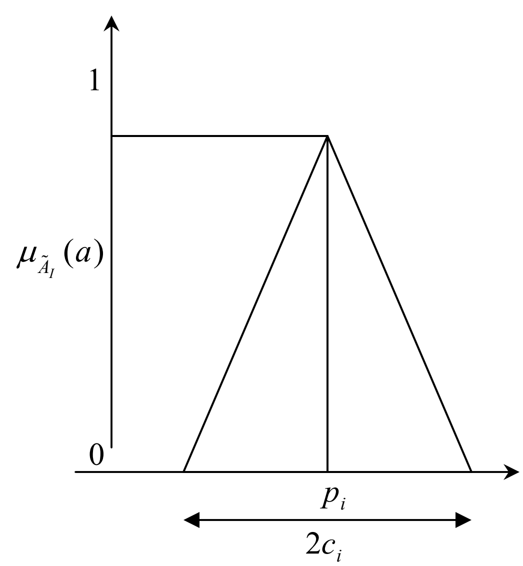

Fig. 2. Membership function for $\tilde{A}_i$

aim of the fuzzy regression problem considered here is to determine a family of such symmetric triangles representing all the coefficients in the FLR formula (equation 3). From this geometric representation, the analytical expression for $\tilde{A}_i$ can be found as:

$$\mu_{\tilde{A}_I}(a_i) = \begin{cases} 1 - \dfrac{|p_i - a_i|}{c_i} \\ 0 \end{cases} \quad (4)$$

The non-zero value occurs when $p_i - c_i \le a_i \le p_i + c_i$, otherwise the value of the membership function remains zero. Thus each coefficient of regression (which is a fuzzy number) is characterized by two crisp values: spread and the mid point of base. Hence one can represent the vector of fuzzy coefficients ($\tilde{A}$), to be made of two vectors: the spread vector (c) and the mid point vector (p), given by:

$$\tilde{A} = \begin{bmatrix} \tilde{A}_0 & \tilde{A}_1 & \tilde{A}_2 & \dots & \tilde{A}_n \end{bmatrix} = \{p \quad c\} \quad (5)$$

where $p = \{p_0 \quad p_1 \quad p_2 \quad \cdots \quad p_n\}$ and $c = \{c_0 \quad c_1 \quad c_2 \quad \dots \quad c_n\}$. Thus the equation (3) can be modified as:

$$\Delta\tilde{\tilde{x}} = (p_0, c_0) + (p_1, c_1)t_1 + \dots + (p_n, c_n)t_n \quad (6)$$

Just like the membership functions of the fuzzy coefficients, the membership function of the fuzzy dependent variable ($\Delta\tilde{\tilde{x}}$) is given by:

$$\mu_{\Delta\tilde{x}}(\Delta x) = \begin{cases} \max(\min_i[\mu_{\tilde{A}_I}(a_i)]) \\ 0 \end{cases} \quad (7)$$

The non-zero value occurs when $\{a \mid \Delta x = f(t,a) \ne \phi\}$, otherwise the value of the membership function remains zero. Substituting (4) in (7), equation (8) can be obtained. Now it can be observed that, the algebraic form of equation (8) is similar to (4), and hence, the membership function for the fuzzy dependent variable, can also be geometrically represented as symmetric triangle, with redefinition of the mid point and spread. The new value of mid point is equal to $\sum_{i=1}^{n} p_i t_i$ and the new spread becomes $\sum_{i=1}^{n} c_i |t_i|$.

$$\mu_{\Delta\tilde{x}}(\Delta x) = \begin{cases} 1 - \dfrac{|\Delta x - \sum_{i=1}^{n} p_i t_i|}{\sum_{i=1}^{n} c_i |t_i|}, & t_i \ne 0 \\ 1 & t_i = 0, \Delta x = 0 \\ 0 & t_i = 0, \Delta x \ne 0 \end{cases} \quad (8)$$

At this point, it is desired to determine the optimum fuzzy coefficients $\tilde{A}_{OPT}$ such that the confidence level in the corresponding dependent variable remains more than a specified threshold value (h), i.e.

$$\mu_{\Delta\tilde{x}_j}(\Delta x_j) \ge h \forall j \in N^+ \quad (9)$$

Here N^+ stands for the set of positive integers. In other words, one needs to find the fuzzy coefficients such that the spread of the fuzzy output is minimized. Tanaka et al [43] formulated the objective function for this optimization problem as:

$$O_f = \min\left\{ mc_0 + \sum_{j=1}^{m}\sum_{i=1}^{n} c_i t_{ij} \right\} \quad (10)$$

where $t_{0j} = 1 \forall j = 1, 2, \dots, m$. This objective function O_f needs to be minimized subjected to two inequality constraints (derived by Tanaka et al [43] from (9)) given by (11) and (12):

$$x_j \ge \sum_{i=0}^{n} p_i t_{ij} - (1-h)\sum_{i=0}^{n} c_i t_{ij} \quad (11)$$

$$x_j \leq \sum_{i=0}^{n} p_i t_{ij} + (1-h)\sum_{i=0}^{n} c_i t_{ij} \qquad (12)$$

Equation (11) and (12) represent total $2m$ constraints. Now this becomes a linear programming problem, which must be solved to find the mid points and spreads of the fuzzy coefficients.

Constrained Optimization with Linear Programming using simplex Algorithm

A code has been written by the authors which solves the aforementioned linear programming problem using simplex algorithm (with artificial variables and big M method). It can be noted that, unlike conventional simplex algorithm, the present algorithm needs to be customized since the fuzzy requirement of equation (9) poses two sided constraint in the form of (11) and (12).

This code results the values of fuzzy coefficients (i.e. the mid points and slopes) for a single straight line segment fit. These values are used in constructing the elements of matrix Q as explained in the next section.

SIMULATION RESULTS

A data window of fixed size (say λ) is constructed and the predicted values of state residuals are passed through this window. A straight line segment is fitted (using FLR) through the predicted state residual values within this window and its slope is found. This slope value is assigned to the derivative of the first state residual value inside the window (to be used for equation (1)). Now the window is shifted by one data position and the same is repeated. It can be noted that, the procedure runs in real time, since to make a good prediction of the next state residual, the driving noise covariance matrix must use equation (1) with the already passed value of slope. This process runs in parallel for all the six state residuals ($\vartriangle x, \vartriangle y, \vartriangle z, \vartriangle \dot{x}, \vartriangle \dot{y}, \vartriangle \dot{z}$). If, during the entire run of the code, total number of predictions for each state residual is Π, then $(\Pi - \lambda + 1)$ slopes are computed for each residual. Total number of slopes is, of course, six times this value.

In this paper, results are presented for simulations performed on PSLV-C1 data. A data window of size 10 (i.e. λ=10) has been used. The values of 'big M's (for the six types of state residuals) are chosen to be 1000 times larger than the average values of the corresponding state residuals, leaving the few starting values because of large initial errors. The sampling time is one second and for the first 10 seconds a Bayesian filter is used. All the results of this fuzzy adaptive state noise driven EKF model is compared with the results of a similar state noise driven EKF with least square straight line fitting for the same data. For comparison purpose, both the simulations were carried out with same size (λ=10) of data window. In all the plots presented in this paper, the solid curves represent the result for the case when driving noise is constructed using least square fit and the dotted curves represent the case when the same is done using FLR.

In Fig. 3 – Fig. 8, orbit parameters are plotted in the order of semimajor axis (a), eccentricity (e), inclination (i), nodal angle (Ω), argument of perigee (ω) and true anomaly (θ). These figures reveal that, though both least square and FLR curves show initial oscillations, for the first four parameters, the FLR curve sharply comes to the converged value, quicker than the least square curve and then settles down with few minute oscillations around that value. In

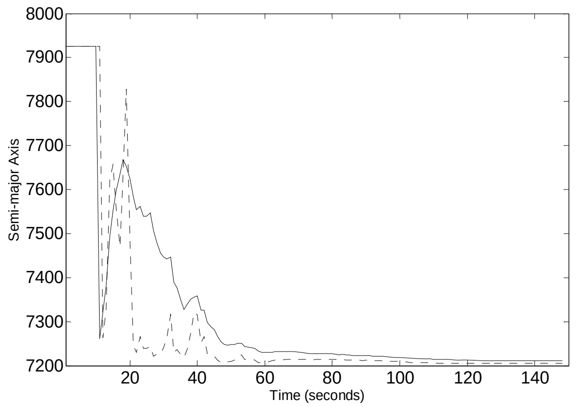

Fig. 3. Semimajor axis (a) estimated by the EKF

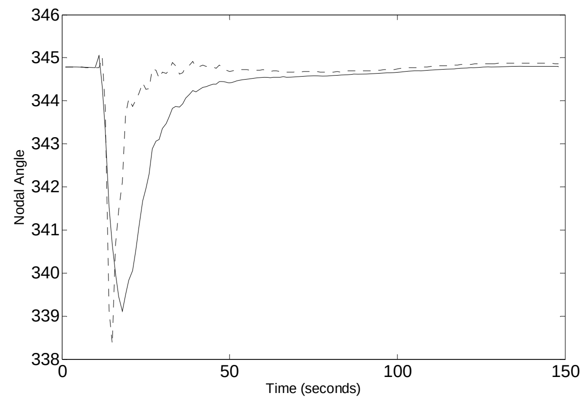

Fig. 6. Nodal angle (Ω) estimated by the EKF

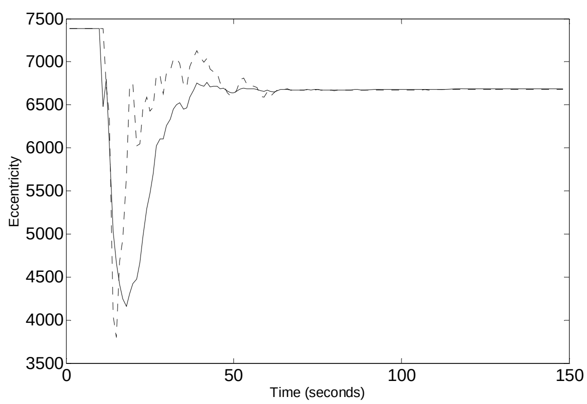

Fig. 4. Eccentricity (e) estimated by the EKF

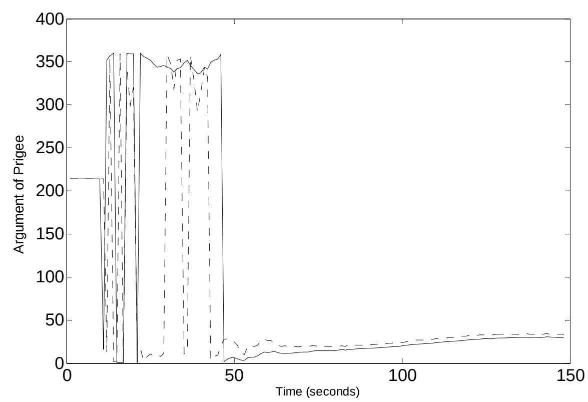

Fig. 7. Argument of perigee (ω) estimated by the EKF

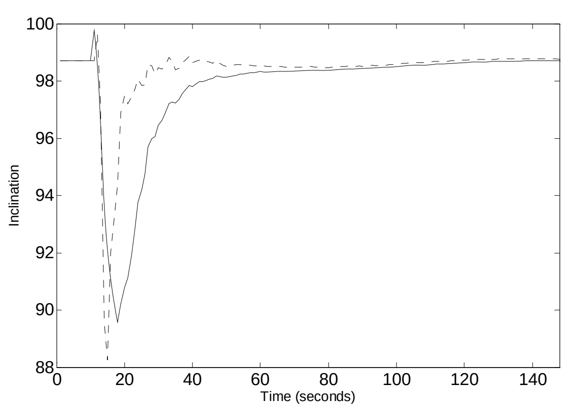

Fig. 5. Inclination (i) estimated by the EKF

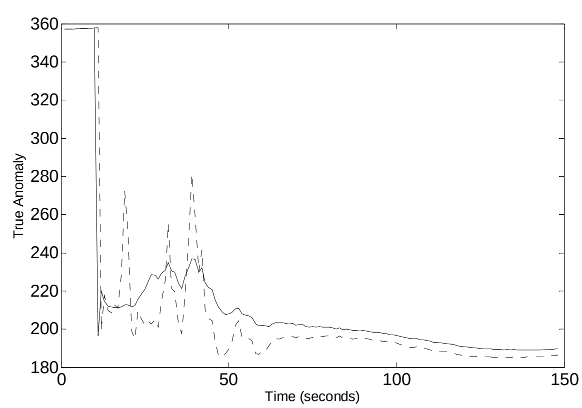

Fig. 8. True anomaly (θ) estimated by the EKF

contrary, the least square curve takes longer to settle, with much greater spread of oscillations.

Particular observation of Fig. 7, at first glance, indicates that for first 50 seconds, the FLR curve undergoes severe oscillation compared to the least square curve. This, however, is a result of the fact that the value of the argument of perigee (ω), in both the cases, remain close to the $360^\circ/0^\circ$ line in the angular plane. In the least square curve, the oscillation remains confined mostly in one side of the dividing line (hence the curve seems more or less steady in the neighborhood of 360°). However, in the FLR curve, since the oscillation of the numerical values often cross this dividing line, seemingly large jumps show up. In Appendix 1, the angular histograms are plotted to show the distribution of values of argument of perigee, for both the least square (Fig. A1) and FLR (Fig. A2) case.

Fig. 9 – Fig. 12 depict the observation residuals in the order of range residual ($\vartriangle\rho$), range rate residual ($\vartriangle\dot{\rho}$), azimuth residual ($\vartriangle Az$) and elevation residual ($\vartriangle El$). Although for range and range rate residuals the results of the least square and FLR curve are almost comparable, in the case of azimuth residual (Fig. 11), the FLR curve has a smaller value of largest overshoot with

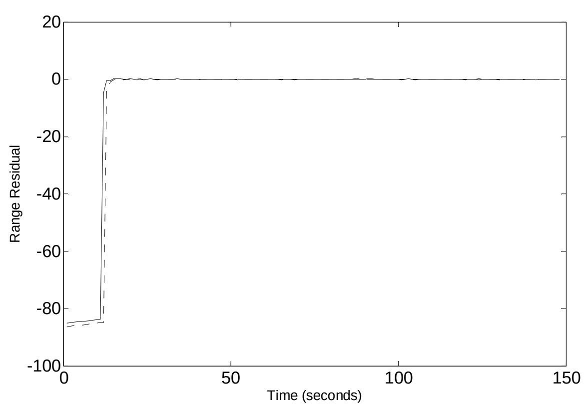

Fig.9. Range residual ($\vartriangle\rho$)

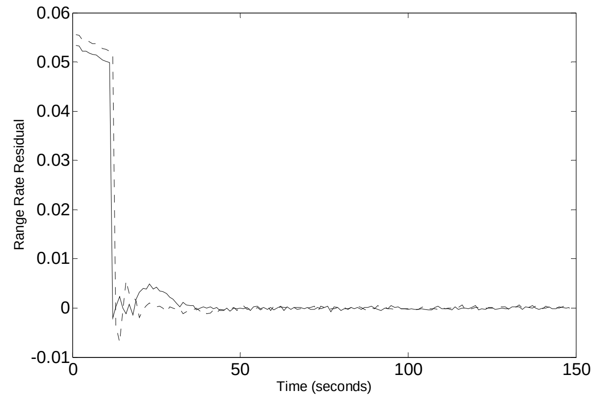

Fig.10. Range rate residual ($\vartriangle\dot{\rho}$)

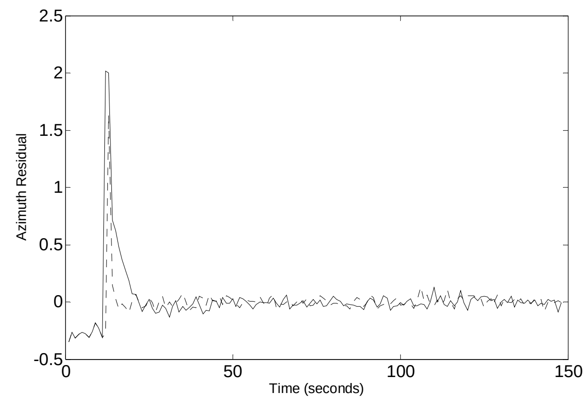

Fig.11. Azimuth residual ($\vartriangle Az$)

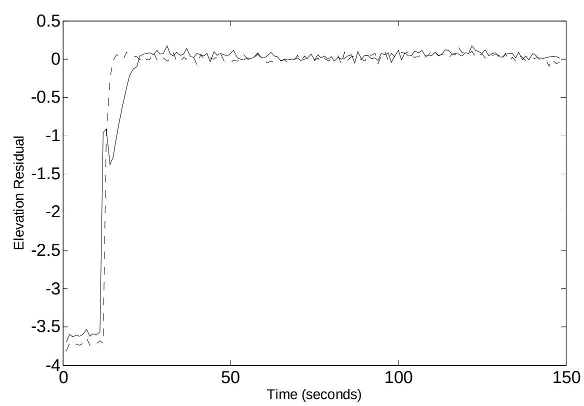

Fig.12. Elevation residual ($\vartriangle El$)

lesser spread. In case of elevation residual (Fig. 12), the FLR curve reaches the minimum value quicker than the least square curve.

CONCLUSION

This paper attempted the fuzzification of the state residual based formulation of process noise covariance matrix, in order to ensure positive definiteness of the state covariance matrix. Fuzzy linear regression (FLR) has been used with a fixed size data window model to compute the slopes of the state residuals and thereby diagonal elements of the driving noise covariance matrix were formulated based on a spring-mass-damper system analogy. The process of finding slopes through a possibilistic fuzzy regression model allowed to capture the inherent ambiguity of the orbit determination process using EKF. Kalman filter being a stochastic estimator, captures the probabilistic aspect of the process. The fuzzy formulation can aid it to reflect the possibilistic nature of the problem as well. The improvement of the results, no matter to what extent, is a testimony for this fact.

The authors, in the retrospect, agree that though the fuzzification of the state residuals for the formulation of driving noise covariance matrix makes the response of the filter quicker, further improvements can be made by having a more elegant formulation of the noise covariance matrix. Moreover, it was felt that, a serious attempt should be made to make the filter more robust. With these considerations in mind, the paper makes an open-ended conclusion.

APPENDIX 1

The angle histograms shown below illustrate the distribution of numerical values of argument of perigee (ω). Each bin holds values within that particular angular range; the length of the bin (shown on radial scale) signifies the number of values present in that bin.

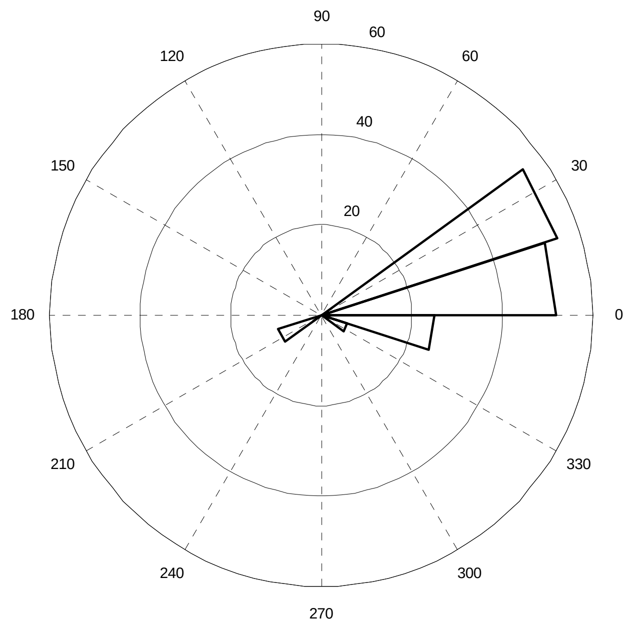

Fig.A1. Angular distribution of (ω): least square case

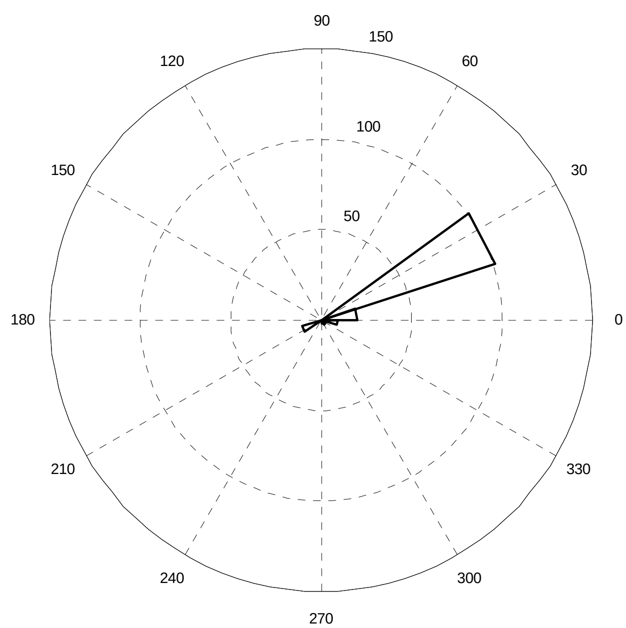

Fig.A2. Angular distribution of (ω): FLR case

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